UNIVERSITY OF COLOMBO, SRI LANKA

UNIVERSITY OF COLOMBO SCHOOL OF COMPUTING

DEGREE OF BACHELOR OF INFORMATION TECHNOLOGY (EXTERNAL)

Academic Year 2004/2005 – 1st Year Examination – Semester 1

IT1102: Mathematics for Computing I

26th February, 2005
(TWO HOURS)

Important Instructions :

- The duration of the paper is 2 (two) hours.
- The medium of instruction and questions is English.
- The paper has **45** questions and **8** pages.
- All questions are of the MCQ (Multiple Choice Questions) type.
- All questions should be answered.
- Each question will have 5 (five) choices with **one or more** correct answers.
- All questions will carry equal marks.
- There will be a penalty for incorrect responses to discourage guessing.
- The mark given for a question will vary from –1 (*All the incorrect choices are marked & no correct choices marked*) to +1 (*All the correct choices are marked & no incorrect choices are marked*).
- Answers should be marked on the special answer sheet provided.
- Note that questions appear on both sides of the paper.
- If a page is not printed, please inform the supervisor immediately.
- Mark the correct choices on the question paper first and then transfer them to the given answer sheet which will be machine marked. **Please completely read and follow the instructions given on the other side of the answer sheet before you shade your correct choices.**

Notations:

Z – set of integers N – set of positive integers
R – set of real numbers ϕ - empty set
S – Universal set

1) If repetitions are not permitted, how many 3-digit even numbers can be formed from the 5 digits 3, 4, 5, 7 and 8?

(a) 24	(b) 60	(c) 75
(d) 30	(e) 50	

2) $n^2 + n - 6$ is equal to

(a)	$\frac{(n+3)!}{(n+1)!}$	(b)	$\frac{(n-2).(n+3)!}{(n+2).(n+1)!}$
(c)	$(n+3)(n-2)$	(d)	$(n+3).(n-2)!$
(e)	$\frac{(n+3)!(n-2)!}{(n+2)!(n-3)!}$		

3) In how many ways can 3 boys and 4 girls sit in a row if all the girls are to sit together?

(a) 4. 4! 3!	(b) 3. 4! 3!	(c) 2. 4! 3!
(d) 4! 3!	(e) 12	

4) How many 4-letter codes can be formed from the English alphabet if repeats are allowed?

(a) $\frac{26!}{22!}$	(b) $(26)^4$	(c) 26. 4!
(d) 4. 26!	(e) 26!	

5) In how many ways can 3 Sri Lankans, 3 Indians and 3 Chinese be seated at a round table so that those of the same nationality sit together?

(a) $(3!)^3$	(b) $2.(3!)^3$	(c) $(3!)^4$
(d) $\frac{9!}{3!3!3!}$	(e) 9!	

6) ${}_{100}C_{98}$ is equal to

(a) 9800	(b) 9900	(c) 4950
(d) 200	(e) 4900	

7) The 4^{th} term of the Binomial expansion of $(10 + 10)^{10}$ is

(a) $\binom{10}{3}10^{10}$	(b) $\binom{10}{7}10^{10}$	(c) $\binom{10}{4}10^{10}$
(d) $\binom{10}{6}10^{10}$	(e) 10^4	

8) How many groups of 4 people can be formed from 6 people?

(a) 15	(b) 30	(c) 60
(d) 120	(e) 45	

9) Suppose we toss 2 distinct dice simultaneously and observe the numbers which appear on top of the 2 dice respectively. How many outcomes does the sample space of this "experiment" consist of?

(a) 6	(b) 12	(c) 36
(d) 18	(e) 22	

10) A coin is tossed 4 times and the face value (H or T) is observed in sequence. The number of elementary outcomes in the sample space is

(a) 8	(b) 4	(c) 2
(d) 16	(e) 1	

11) How many distinct permutations can be formed out of the letters of the word "STATISTICS"?

(a) $\frac{10!}{3!3!2!}$	(b) $\frac{10!}{3!}$	(c) $\frac{10!}{3!3!}$
(d) 10!	(e) $3 \times 10!$	

12) The number of different ordered samples of size r without replacement from a population of (r + 2) objects is equal to

(a) (r + 2)!	(b) $\frac{(r+2)!}{r!}$	(c) $\frac{(r+2)!}{2!}$
(d) r!	(e) 2(r!)	

13) The ratio of the number of different ordered samples of size 2 without replacement, to that with replacement from a box of 10 objects is

(a) 8:10	(b) 7:10	(c) 7:9
(d) 3:4	(e) 10:7	

14) Suppose we toss 2 distinct dice simultaneously and observe the numbers which appear on top of the 2 dice respectively. How many elementary outcomes does the sample space of this "experiment" consists of ?

(a) 6	(b) 12	(c) 36
(d) 18	(e) 24	

15) Let A and B be mutually exclusive events with $P(A^c \cap B^c) = \frac{3}{8}$ and $P(A^c) = \frac{5}{8}$.

Then P(B) =

(a) $\frac{3}{8}$	(b) $\frac{1}{4}$	(c) $\frac{1}{8}$
(d) $\frac{5}{8}$	(e) 1	

16) $A = \{(x, y) \mid x, y \in R \text{ and } x^2 + y^2 = 17\}$ and $B = \{(x, y) \mid x, y \in R \text{ and } x + y = 5\}$. Then, $A \cap B$ equals

(a) {4}	(b) {1, 4}	(c) {(1, 4)}
(d) {(4, 1)}	(e) {(1, 4), (4, 1)}	

17) A, B are subsets of a universal set S and $C = A \setminus B$. Which of the following are/is true?

(a) C is a subset of A.	(b) C is a subset of B.	(c) $C \cap B = \phi$.
(d) $A \cap C \cap B^c = \phi$.	(e) $A \cap B^c \cap C = C$.	

18) $A = \{2, 10, 8, 4, 6\}$ and $B = \{3, 4, 6, 9, 10\}$.
Consider the following:
(i) $A \cup B = \{2, 3, 4, 5, 6, 8, 9, 10\}$
(ii) $A \cap B = \{4, 6, 10\}$
(iii) $A \setminus B = \{2, 8, 6\}$.

Then

(a) only (i) and (ii) are true.	(b) only (ii) is true.	(c) only (iii) is true.
(d) only (ii) and (iii) are true.	(e) all of (i), (ii), (iii) are true.	

19) The sets A, B and C are such that $A \cap B$ is a subset of C, C is a subset of $A \cup B$, $4 \in A$, $5 \in C$, $6 \in B$ and $4 \in B$.

Which of the following are/is true?

(a) $4 \in C$	(b) $5 \in A$	(c) $6 \in A$
(d) $5 \in B$	(e) $6 \in C$	

20) For any sets P and Q, $P \subseteq Q$ denotes 'P is a subset of Q'. The sets A, B, C are such that $A \cup B \subseteq C$, $B \cup C \subseteq A$, $A \cup C \subseteq B$, $3 \in A$, $5 \in B$ and $2 \in C$. Which of the following are/is true?

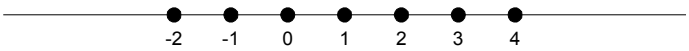
(a) $3 \in B$	(b) $5 \in C$	(c) $2 \in B$
(d) $2 \in A$	(e) $3 \in C$	

21) $A = \{x \mid x \in Z \text{ and } x \text{ is even}\}$,
$B = \{x \mid \text{There is } n \in N \text{ such that } x = n2\}$,
$C = \{x \mid x \in R \text{ and } 1 < x < 2\}$,
$D = \{x \mid x \in N \text{ and } x \text{ is odd and } 1 < x < 36\}$ and
$E = \{x \mid x \in Z \text{ and } x < -6\}$.

Among the sets A, B, C, D and E, the infinite sets are only:

(a) A, B, C	(b) A, B, C, D, E	(c) A, B, C, E
(d) A, C, E	(e) A, C	

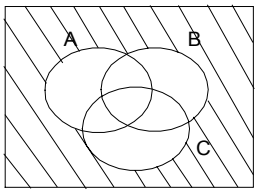
22) All the members of a set A are indicated by dots on the number line as given below.



The set A is equal to

(a) $\{x \mid x \in Z \text{ and } -2 < x < 4\}$.	(b) $\{x \mid x \in Z \text{ and } -2 \le x \le 4\}$.
(c) $\{x \mid x \in Z \text{ and } -4 < x < 5\}$.	(d) $\{x \mid x \in Z \text{ and } -2 < x < 5\}$.
(e) $\{x \mid x \in Z \text{ and } -2 \le x < 5\}$.	

23) Consider the following Venn diagram.



The shaded portion in the Venn diagram represents

(a) $(A^c \cap B^c) \cup (B^c \cap C^c) \cup (C^c \cap A^c)$.	(b) $A^c \cup B^c \cup C^c$.
(c) $A^c \cap B^c \cap C^c$.	(d) $(A \cap B^c) \cup (B \cap C^c) \cup (C \cap A^c)$.
(e) $(A^c \cap C^c) \cup (B^c \cap C^c)$.	

24) Consider the following statements:

(i) If I drink tea, then I work hard.
(ii) If I am going to sleep, then I do not drink tea.
(iii) If I work hard, then I am not going to sleep.

Now:

'I drink tea' is denoted by one of p, not p, q, not q, r, not r.

'I work hard' is denoted by one of p, not p, q, not q, r, not r.

'I am going to sleep' is denoted by one of p, not p, q, not q, r, not r.

Which of the following could be correct?

(a) (i) means $r \Rightarrow p$	(ii) means $p \Rightarrow$ not q	(iii) means $q \Rightarrow$ not r
(b) (i) means $p \Rightarrow$ not q	(ii) means $r \Rightarrow p$	(iii) means $q \Rightarrow r$
(c) (i) means $p \Rightarrow$ not q	(ii) means $r \Rightarrow$ not p	(iii) means $q \Rightarrow$ not r
(d) (i) means $p \Rightarrow q$	(ii) means $r \Rightarrow$ not p	(iii) means $q \Rightarrow$ not r
(e) (i) means $p \Rightarrow$ not q	(ii) means $r \Rightarrow p$	(iii) means $q \Rightarrow$ not r

25) Consider the following:

(i) $x > y$. (ii) $x > 4$. (iii) $5 > 4$. (iv) Given any real number x, $x > y$.

Which of the following are/is correct?

(a) (i), (ii), (iii) and (iv) are all propositions.
(b) (i), (ii), (iii) and (iv) are all not propositions.
(c) (iii) is a proposition.
(d) (ii) is a proposition.
(e) (iv) is a proposition.

26) Denote $(p \vee (\text{not } q) \vee r) \wedge ((\text{not } p) \vee q \vee r) \wedge (p \vee q \vee (\text{not } r))$ by β.

Which of the following are/is correct?

(a) β is false when p is false, q is true and r is false.
(b) β is false when p is false, q is false and r is false.
(c) β is false when p is true, q is true and r is false.
(d) β is false when p is true, q is true and r is true.
(e) β is false when p is false, q is false and r is true.

27) Consider the following truth tables:

(i)

p	q	$p \Rightarrow (q \Rightarrow p)$
T	T	T
T	F	T
F	T	T
F	F	T

(ii)

p	q	$(p \Rightarrow (p \wedge q)) \vee ((p \wedge q) \Rightarrow p)$
T	T	T
T	F	T
F	T	T
F	F	T

(iii)

p	q	$(p \Rightarrow q) \Rightarrow (q \Rightarrow p)$
T	T	T
T	F	T
F	T	T
F	F	T

Which of the following is/are correct?

(a) (i), (ii) and (iii) are all correct.	(b) (i) is correct but (ii) and (iii) are incorrect.
(c) (i), (ii) are correct but (iii) is incorrect.	(d) (i), (ii) and (iii) are all incorrect.
(e) (i) and (iii) are correct but (ii) is incorrect.	

28) Consider the following truth table

p	q	r	β_1	β_2	β_3
T	T	T	T	T	T
T	F	T	T	T	T
F	T	T	T	T	T
F	F	T	T	T	T
T	T	F	T	F	F
T	F	F	F	T	T
F	T	F	T	T	F
F	F	F	F	T	F

Now, $p \Rightarrow (q \Rightarrow r)$ is one of $\beta_1, \beta_2, \beta_3$ and $(p \Rightarrow q) \Rightarrow r$ is also one of $\beta_1, \beta_2, \beta_3$.
Which of the following is/are correct?

(a) $p \Rightarrow (q \Rightarrow r)$ is φ_2 and $(p \Rightarrow q) \Rightarrow r$ is φ_1
(b) $p \Rightarrow (q \Rightarrow r)$ is φ_1 and $(p \Rightarrow q) \Rightarrow r$ is φ_2
(c) $p \Rightarrow (q \Rightarrow r)$ is φ_1 and $(p \Rightarrow q) \Rightarrow r$ is φ_3
(d) $p \Rightarrow (q \Rightarrow r)$ is φ_3 and $(p \Rightarrow q) \Rightarrow r$ is φ_1
(e) $p \Rightarrow (q \Rightarrow r)$ is φ_2 and $(p \Rightarrow q) \Rightarrow r$ is φ_3

29) Out of the 16 possible sets of truth values for p, q, r, s,
$(p \vee q) \Rightarrow (r \wedge s)$ is <u>true only for</u>

(a) 9 sets of values. (b) 12 sets of values. (c) 8 sets of values.
(d) 7 sets of values. (e) 6 sets of values.

30) Consider the following propositions.

(i) $(\text{not } p) \wedge q$ (ii) $\text{not } (p \wedge q)$ (iii) $(\text{not } p) \vee q$
(i) $\text{not } (p \wedge (\text{not } q))$ (v) $p \wedge (q \vee r)$ (vi) $(p \wedge q) \vee r$

Which of the following are(is) correct?

(a) (i), (ii) are equivalent. (b) (iii), (iv) are equivalent. (c) (v), (vi) are equivalent.
(d) (ii), (iii) are equivalent. (e) (ii), (iv) are equivalent.

31) It is given that $p \Leftrightarrow q$ and $q \Leftrightarrow r$ are true. <u>Then</u> which of the following must necessarily be true?

(a) $p \Rightarrow r$ (b) $p \Leftrightarrow r$ (c) p
(d) q (e) $p \vee q$

32) Consider the following:
(i) For every positive integer n, $2^n \geq n^2$.
(ii) For every real number x, $x^4 = 1 \Rightarrow x = 1$.
(iii) For every positive integer n, $n^2 = 4 \Rightarrow n = 2$.

Which of the following are/is correct?

(a) (i) is true. (b) (ii) is true. (c) (iii) is true.
(d) (i) and (ii) are both false. (e) (ii) and (iii) are both false.

33) Let the universal set be N. Now consider the following.
(i) $\forall x, x^2 + x \geq 2 \wedge x^2 \geq 1$ (ii) $\forall x, x^2 + x > 3 \vee x^2 + x = 2$ (iii) $(\forall x, x^2 + x > 3) \vee (\forall x, x^2 + x = 2)$

Which of the following is/are correct?

(a) (i), (ii) and (iii) are all true.	(b) (i), (ii) are true but (iii) is false.
(c) (i), (iii) are true but (ii) is false.	(d) (i) is true but (ii), (iii) are false.
(e) (i), (ii) and (iii) are all false.	

34) Let the universal set be Z. Now consider the following:
(i) $\forall x, x > 1 \vee x < 6$ (ii) $(\forall x, x > 1) \vee (\forall x, x < 6)$ (iii) $\exists x, x > 1 \wedge x < 6$.

Which of the following are/is correct?

(a) (i) is true.	(b) (ii) is true.	(c) (iii) is true.
(d) Both (i) and (ii) are true.	(e) Both (i) and (iii) are true.	

35) For $n \in N$, P(n) is a proposition such that for any $n \in N$, $P(n + 1) \Rightarrow P(n)$ is true.

Which of the following are/is true?

(a) For any $n \in N$, P(n)	(b) For any $n \in N$, $P(1) \Rightarrow P(n)$
(c) For any $n \in N$, $(P(5) \wedge n < 5) \Rightarrow P(n)$	(d) $P(10) \Rightarrow P(9)$
(e)) $P(9) \Rightarrow P(10)$	

36) Let A = {1,2} and B = {a,b}. Which of the following sets is/are equal to $A \times B$?

(a) {{1,a},{1,b},{2,a},{2,b}}	(b) {(a,1),(b,1),(a,2),(b,2)}
(c) {{a,1},{b,1},{a,2},{b,2}}	(d) {(1,a),(1,b),(2,a),(2,b)}
(e) { (1,2), (a,b) }	

37) Let R be a relation from the set A to set B. Which of the following is/are correct?

(a) The domain of R is always A.
(b) The domain of R is a subset of A.
(c) The domain of R is a subset of A consisting of the second elements of the ordered pairs of R.
(d) The domain of R is a subset of A consisting of the first elements of the ordered pairs of R.
(e) The domain of R is a subset of B.

38) Let R be the relation on {1,2,3,4} defined by "x is less than y". Then R is

(a) {(1,1),(1,2),(1,3),(1,4),(2,3),(2,4),(3,4)}	(b) {(1,2),(1,3),(1,4),(2,3),(2,4),(3,4)}
(c) {(1,1),(2,2),(3,3),(4,4)}	(d) {(2,1),(3,1),(4,1),(3,2),(4,2),(4,3)}
(e) {{1,2},{1,3},{1,4},{2,3},{2,4},{3,4}}	

39) Let W = {a,b,c,d}. Which of the following sets of ordered pairs is/are function(s) from W into W?

(a) {(b,a),(c,d),(d,a),(c,d),(b,d)}	(b) {(a,a),(b,a),(a,b),(c,d),(d,a)}
(c) {(b,a),(c,d),(d,a),(c,d),(a,d)}	(d) {(a,b),(b,b),(c,b),(d,b)}
(e) {(a,a),(b,b),(c,c),(d,d)}	

40) Consider the following functions.
(i) To each person on the earth assign his/her age in years.
(ii) To each country in the world assign the latitude and longitude of its capital. (Assume that a country can have only one capital).
(iii) To each book written by a single author, assign the author.
(iv) To each country in the world where there is a prime minister, assign the prime minister.

Which of the above functions is/are one-to-one?

(a) (i)	(b) (ii)	(c) (iii)
(d) (iv)	(e) (i), (ii), (iii), (iv)	

Questions 41) and 42) are based on the following:

Let <B,+,*,',0,1> be a Boolean Algebra, where B is a set , + and * are the sum and product operations respectively defined on the elements of B, 0 and 1 are the zero and unit elements respectively, ' is the complement (negation) operator and an any element in B.

41) Which of the following is/are correct?

(a) if a + x = 1 and a * x = 0 then x = a'
(b) a' = a' + 0
(c) (a')' = a
(d) 1' = 1
(e) 0 = a' + 0

42) The dual of the Boolean equation
(a * 1) * (0 + a') = 0 is

(a) (1 * a) * (a' + 0) = 0	(b) (a * 0) * (1 + a') = 1
(c) (a + 0) + (1 + a') = 1	(d) (a * 1) * (0 + a') = 1
(e) (a + 1) + (0 * a') = 0	

43) Let the operator ! be defined on non-negative integers as below.
0! = 1
n! = n × (n-1)! where n is a positive integer.

Then 4! =

(a) 4 × 3 × 2 × 1 × 1	(b) 4 × 3	(c) 4 × 3!
(d) 24	(e) 4 × 0!	

44) $\log_2\left(\frac{1}{16}\right) =$

(a) 4	(b) 2	(c) 8
(d) -4	(e) -2	

45) $125^{\left(\frac{2}{3}\right)} =$

(a) 5	(b) 125	(c) -125
(d) 25	(e) -5	

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