UNIVERSITY OF COLOMBO, SRI LANKA

UNIVERSITY OF COLOMBO SCHOOL OF COMPUTING

DEGREE OF BACHELOR OF INFORMATION TECHNOLOGY (EXTERNAL)

Academic Year 2006 /2007 – 1st Year Examination – Semester 2

IT2103 - Mathematics for Computing I

11th August 2007

(TWO HOURS)

Important Instructions :

- The duration of the paper is 2 (two) hours.
- The medium of instruction and questions is English.
- The paper has **45** questions and **10** pages.
- All questions are of the MCQ (Multiple Choice Questions) type.
- All questions should be answered.
- Each question will have 5 (five) choices with **one or more** correct answers.
- All questions will carry equal marks.
- There will be a penalty for incorrect responses to discourage guessing.
- The mark given for a question will vary from 0 to +1 (*All the correct choices are marked & no incorrect choices are marked*).
- Answers should be marked on the special answer sheet provided.
- Note that questions appear on both sides of the paper.
- If a page is not printed, please inform the supervisor immediately.
- Mark the correct choices on the question paper first and then transfer them to the given answer sheet which will be machine marked. **Please completely read and follow the instructions given on the other side of the answer sheet before you shade your correct choices.**

Notations:

Z – set of integers	N – set of positive integers
R – set of real numbers	ϕ - (null) empty set
S – Universal set	R^+- set of non-negative real numbers

1) Let a be any positive number and p, q be any two integers where $q \neq 0$. Which of the following correctly interpret $a^{\frac{p}{q}}$?

(a) $\sqrt[p]{a^q}$

(b) $\sqrt[q]{a^p}$

(c) $\dfrac{1}{a^{\frac{q}{p}}}$

(d) $\underbrace{\sqrt[q]{a} \times \sqrt[q]{a} \times \cdots\cdots\cdots\cdots\cdots\cdots \times \sqrt[q]{a}}_{\text{p-times}}$

(e) $\underbrace{\sqrt[p]{a} \times \sqrt[p]{a} \times \cdots\cdots\cdots\cdots\cdots\cdots \times \sqrt[p]{a}}_{q\text{-times}}$

2) $2\log_{10} 5 + \log_{10} 4$ is equal to

(a) $\log_{10} 25 + 2\log_{10} 2$

(b) $2(\log_{10} 5 + \log_{10} 2)$

(c) $2\log_{10}(5+4)$

(d) 2

(e) $2\log_{10}(5\times 4)$

3) $A = \{(m, n) \mid m, n \in Z \text{ and } n < m < n + 1\}$.
A equals :

(a) $\{(0, 0)\}$
(b) $\{\phi\}$
(c) ϕ
(d) $\{0\}$
(e) $\{x \mid x \in Z \text{ and } x^2 = -1\}$

4) $A = \{(x, y) \mid x, y \in R \text{ and } 2x + 3y = 13\}$
$B = \{(x, y) \mid x, y \in R \text{ and } 3x + 2y = 12\}$

$A \cap B$ equals

(a) $\{8, -1\}$
(b) $\{-1, 5\}$
(c) $\{(-1, 5)\}$
(d) $\{(2, 3)\}$
(e) $\{2, 3\}$

5) $A = \{-2, -1, 0\} \cup N$ and $B = \{2, 1, 0\} \cup \{-n \mid n \in N\}$.

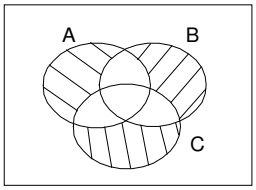
Which of the following are/is true?

(a) $A \cup B = A$ and $A \setminus B = \{n \mid n \in N \text{ and } n \geq 3\}$.
(b) $A \cup B = Z$ and $A \setminus B = \{n \mid n \in N \text{ and } n \geq 3\}$
(c) $A \cup B = Z$ and $A \setminus B = N$
(d) $A \cup B = A$ and $A \setminus B = \{-n \mid n \in N \text{ and } n \geq 3\}$
(e) $A \cup B = Z$ and $A \setminus B = N \cup \{0\}$

6) The sets A, B, C are such that $A \setminus B = A \setminus C$. Which of the following is/are true?

(a) $A \cap B = A \cap C$ (b) $A \cup B = A \cup C$ (c) $B = C$
(d) $A \cap B$ is a subset of A (e) $A \cap B$ is a subset of C

7)



The shaded portion in the Venn diagram represents

(a) $(A \cap B \cap C)^c$. (b) $(A \cap B^c) \cup (B \cap C^c) \cup (C \cap A^c)$.
(c) $(A \cup B)^c \cup (B \cup C)^c \cup (C \cup A)^c$. (d) $(A \cap B)^c \cup (B \cap C)^c \cup (C \cap A)^c$.
(e) $(A \cap B^c \cap C^c) \cup (B \cap C^c \cap A^c) \cup (C \cap B^c \cap A^c)$.

8) Consider the following truth table.

p	q	r	u_1	u_2	u_3
T	T	T	T	T	T
T	T	F	F	F	F
T	F	T	F	F	F
T	F	F	F	F	F
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	T
F	F	F	T	F	F

Now, $(p \Rightarrow q) \wedge r$ is one of u_1, u_2, u_3 and $p \Rightarrow (q \wedge r)$ is also one of u_1, u_2, u_3.
Which **one** of the following is correct?

(a) $(p \Rightarrow q) \wedge r$ is u_1 and $p \Rightarrow (q \wedge r)$ is u_2. (b) $(p \Rightarrow q) \wedge r$ is u_2 and $p \Rightarrow (q \wedge r)$ is u_1.
(c) $(p \Rightarrow q) \wedge r$ is u_2 and $p \Rightarrow (q \wedge r)$ is u_3. (d) $(p \Rightarrow q) \wedge r$ is u_1 and $p \Rightarrow (q \wedge r)$ is u_3.
(e) $(p \Rightarrow q) \wedge r$ is u_3 and $p \Rightarrow (q \wedge r)$ is u_2.

9) Out of the 16 possible sets of truth values for p, q, r, s,
$(p \wedge (\text{not } q) \wedge (\text{not } r) \wedge s) \vee (p \wedge q \wedge r \wedge (\text{not } s)) \vee (p \wedge q \wedge r \wedge s) \vee ((\text{not } p) \wedge q \wedge r \wedge s)$ is **false only for**

(a) 15 sets of values.	(b) 8 sets of values.
(c) 4 sets of values.	(d) 6 sets of values.
(e) 12 sets of values.	

10) It is given that $p \Rightarrow q$, $q \Rightarrow r$, $r \Rightarrow s$ and $s \Rightarrow q$ are true. **Then** which of the following must necessarily be true?

(a) $p \Rightarrow r$	(b) $q \Leftrightarrow r$	(c) $q \Rightarrow p$
(d) $q \Leftrightarrow p$	(e) $p \Leftrightarrow r$	

11) For $n \in N$, P(n) is a proposition such that for any $n \in N$, $P(2n) \Rightarrow P(2n + 2)$ is true.

Which of the following are/is true?

(a) For any $n \in N$, P(n).	(b) For any $n \in N$, $P(2) \Rightarrow P(2n)$.
(c) For any $n \in N$, $(P(1) \wedge P(2)) \Rightarrow P(2n)$.	(d) For any $n \in N$, $P(2) \Rightarrow P(n)$.
(e) For any $n \in N$, $(P(1) \wedge P(2)) \Rightarrow P(n)$.	

12) Consider the following:
(i) The word "**Examination**" has 3 vowels.
(ii) Come here!
(iii) Given any real number x, there is a real number y such that $y > x$.
(iv) $y > x$.

Which of the following are/is correct?

(a) (i) is a proposition.	(b) (ii) is a proposition.	(c) (iii) is a proposition.
(d) (iv) is a proposition.	(e) only (i) is a proposition.	

13) Which of the following sentence(s) is/are propositions?

(a) Colombo is the capital of India.	(b) When is your examination?
(c) Go out please.	(d) $2 + 2 = 4$.
(e) This sentence is false.	

14) Let **N** be the proposition "**Nimal will win the prize**" and **K** be the proposition "**Kamala will win the prize**". Then the conversion of the proposition "**Either Nimal or Kamala will win the price, but not both**" to prepositional calculus will results in the following form(s).

(a) $(N \vee K) \wedge \text{not } (N \vee K)$	(b) $(N \vee K) \vee \text{not } (N \wedge K)$
(c) $(N \vee K) \wedge \text{not } (N \wedge K)$	(d) $(N \vee K) \wedge (N \wedge K)$
(e) $(N \vee K) \wedge (\text{not } N \vee \text{ not } K)$	

15) Consider the following propositions.

(i) If Nimal took the book then it is in his bag.

(ii) If the book is big then it is not in his bag.

If (i) and (ii) are assumed to be true which of the following conclusions can be derived from them?

(a) Nimal took the book.	(b) Nimal did not take the book.
(c) The book is in Nimal's bag.	(d) If the book is big then Nimal did not take it.
(e) If Nimal took the book then it is not big.	

16) Let P and Q be two propositions. If the truth value of P is "true" and the truth value of Q is "false", which of the following is/are true?

(a) The truth value of $P \vee Q$ is "true".	(b) The truth value of $P \wedge Q$ is "true".
(c) The truth value of $P \Rightarrow Q$ is "true".	(d) The truth value of $P \Rightarrow Q$ is "false".
(e) The truth value of $Q \Rightarrow P$ is "true".	

17) Let P be a proposition. Which of the following is/are correct?

(a) $P \vee$ not P is a Tautology.	(b) $P \vee$ not P is a Contradiction.
(c) $P \wedge$ not P is a Tautology.	(d) $P \wedge$ not P is a Contradiction.
(e) $P \Rightarrow P$ is a Tautology.	

18) Let Z, the set of integers be the universal set. Now consider the following.

(i) $\exists x, 3x > 1 \wedge 3x < 1$. (ii) $(\exists x, 3x > 1) \wedge (\exists x, 3x < 1)$ (iii) $(\exists x, 3x > 1) \vee (\exists x, 3x < 1)$

Which of the following are/is correct?

(a) (i) is true.	(b) (ii) is true.
(c) (iii) is true.	(d) (ii) is false.
(e) (iii) is false.	

19) Let N, the set of positive integers be the universal set. Now consider the following.

(i) $\forall x, x^2 + x \geq 2 \wedge x^2 \geq 1$ (ii) $\forall x, x^2 + x > 3 \vee x^2 + x = 2$ (iii) $(\forall x, x^2 + x > 2) \vee (\forall x, x^2 + x = 2)$

Which of the following is/are correct?

(a) (i) is true.	(b) (ii) is true.
(c) (iii) is true.	(d) (ii) is false.
(e) (iii) is false.	

20) Let P, Q, R be three propositions. Which of the following is/are correct?

(a) $P \wedge (Q \wedge R)$ is logically equivalent to $P \wedge Q \wedge R$.
(b) $P \vee (Q \vee R)$ is logically equivalent to $(P \vee Q) \vee R$.
(c) $P \vee (Q \wedge R)$ is logically equivalent to $(P \wedge Q) \vee (P \wedge R)$.
(d) $P \vee (Q \wedge R)$ is logically equivalent to $(P \vee Q) \wedge (P \vee R)$.
(e) $P \Rightarrow Q$ is logically equivalent to $Q \Rightarrow$ not P.

Question 21 – 24 are based on the following definitions.
Let ρ, σ, α and β be four relations defined in Z as below.

$\rho = \{ (a, b) | a \leq b \wedge a, b \in Z \}$
$\sigma = \{ (a, b) | a \geq b \wedge a, b \in Z \}$
$\alpha = \{ (a, b) | a^2 = b^2 \wedge a, b \in Z \}$
$\beta = \{ (a, b) | b = a + 2 \wedge a, b \in Z \}$

21) Which of the following define the relation $\rho \cup \sigma$?

(a) $\{(a, b) \mid a, b \in Z\}$	(b) $\{(a, b) \mid a, b \in Z \wedge (a < b) \wedge (a > b)\}$
(c) $\{(a, b) \mid a, b \in Z \wedge a = b\}$	(d) Z x Z
(e) $\{(a, a) \mid a \in Z\}$	

22) Which of the following define the relation ρ o β ?

(a) $\{(a, b) \mid a, b \in Z \wedge a+2 \leq b\}$	(b) $\{(a, b) \mid a, b \in Z \wedge a+2 \geq b\}$
(c) $\{(a, b) \mid a, b \in Z \wedge b+2 \leq a\}$	(d) $\{(a, b) \mid a, b \in Z \wedge b+2 \geq a\}$
(e) ρ	

23) Which of the following is/are true?

(a) σ and α are reflexive.	(b) σ and α are transitive.
(c) σ and α are symmetric.	(d) σ and β are not transitive.
(e) σ and β are not symmetric	

24) $[3]_\alpha$ is equal to

(a) { 9 }.	(b) { 3 }.	(c) { 3, -3 }.
(d) { 9, -9 }.	(e) { 3, 9 }.	

25) Let A={1,2} and B={4,5}. Which of the following mappings define functions f from A **into** B?

(a) D(f)={1,2} and $f(1)=4, f(2)=5$	(b) D(f)={1} and $f(1)=4, f(1)=5$
(c) D(f)={1,2} and $f(1)=5, f(2)=5$	(d) D(f)={2} and $f(2)=5$
(e) D(f)={1,2} and $f(1)=4, f(1)=4$	

26) Let A={1,4,7}, B={1,4} and f be any function from A into B. Which of the following define functions f which satisfy the condition $f \circ f = f$?

(a) D(f)={1,4,7} and $f(1)=1, f(4)=4, f(7)=4$	(b) D(f)={1,4,7} and $f(1)=1, f(4)=1, f(7)=4$
(c) D(f)={1,4,7} and $f(1)=1, f(4)=4, f(7)=1$	(d) D(f)={1,4,7} and $f(1)=4, f(4)=4, f(7)=1$
(e) D(f)={1,4,7} and $f(1)=1, f(4)=1, f(7)=1$	

27) Suppose B is a non-empty set and A = { $f \mid f$ is bijection from B to B }. Let f , g $\in$ A. Which of the following are true?

(a) $f \circ g \in$ A	(b) $f \circ g = g \circ f$	(c) R(f) $\subseteq$ B
(d) $f^{-1} \in$ A	(e) R(f) = B	

28) Let f be a function defined by D(f)={x | x $\in$ R $\wedge$ x $\neq \frac{1}{2}$ } and $f(x)=\frac{x+1}{2x-1}$ for $\forall$ x $\in$ D(f). Which of the following is/are true?

(a) f is one-one and D(f) = D(f^{-1}).

(b) D(f^{-1})={x | x $\in$ R $\wedge$ x $\neq \frac{1}{2}$ } and $f^{-1}(x)=\frac{1+x}{2x-1}$ for $\forall$ x $\in$ D(f^{-1})

(c) D(f^{-1})={x | x $\in$ R $\wedge$ x $\neq 1$ } and $f^{-1}(x)=\frac{2x-1}{x-1}$ for $\forall$ x $\in$ D(f^{-1})

(d) f is not one-one.

(e) f is one-one and D(f) $\neq$ D(f^{-1}).

29) Let $f(x) = 2x - 1$ and $g(x) = 3x + 1$. Which of the following is/are correct?

(a) $f \circ g\ (x) = 6x + 1$	(b) $f \circ g\ (x) = 6x - 2$
(c) $g \circ f\ (x) = 2(3x - 1)$	(d) $g \circ f\ (x) = 6x - 1$
(e) $f \circ g\ (x) = g \circ f\ (x)$	

30) The statement that "number of permutations of n objects taken r at a time is equal to the number of permutations of n objects taken (n – r) at a time" is

(a) always true.	(b) true only if n = 2r.	(c) true only if n is even.
(d) true only if r = n.	(e) never true.	

31) How many distinct permutations can be formed from all the letters of the word "SMILE"?

(a) 5	(b) $5\times4\times3\times2\times1$	(c) 120
(d) 20	(e) 60	

32) How many distinct permutations can be formed from the letters of the word "CLASSICS"?

(a) $\frac{8!}{2!\times3!}$	(b) $\frac{8!}{2\times3}$	(c) $\frac{8!}{2}$
(d) 8!	(e) $\frac{8!}{3}$	

33) Suppose an item code has 6 places of which the first 2 are selected from the 3 letters A, B, C with repetitions allowed and the last 4 are selected from the 10 digits 0 – 9 with repetitions allowed. The total number of codes will be

(a) $\frac{10!}{5!}$	(b) 6,000	(c) $\frac{9.10!}{6!}$
(d) 90,000	(e) 10,000	

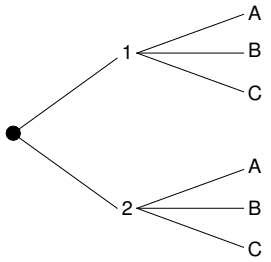
34) In how many ways can 3 Sri Lankans, 3 Indians and 3 Chinese be seated in a row so that those of the same nationality sit together?

(a) $(3!)^4$	(b) $\frac{3!}{4}$	(c) $\frac{9!}{3!3!3!}$
(d) 4(3!)	(e) 3(4!)	

35) The combinations of the letters A, B, C, D taken 3 at a time are

(a) (A, B, C), (A, B, D), (A, C, D), (B, C, D)	(b) (B, A, C), (A, B, D), (C, A, D), (B, C, D)
(c) (C, A, B), (B, A, D), (C, D, A), (C, B, D)	(d) (A, B, C), (B, A, D), (C, A, D), (B, D, C)
(e) (C, B, A), (D, B, A), (D, C, A), (D, A, B)	

36) Consider the following tree diagram.



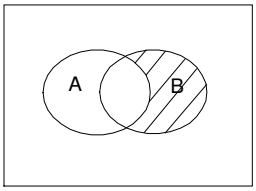
It shows the possible outcomes of the product set

(a) A x B x C where A = {1, 2}.	(b) A x B x C where each set = {1, 2}.
(c) {1, 2} x {A, B, C}.	(d) {A, B, C} x {A, B, C}.
(e) {1,2}.	

37) Suppose we toss 2 dice simultaneously and observe the numbers which appear on top of the 2 dice. How many elementary outcomes does the sample space of this "experiment" consist of?

(a) 6	(b) 12	(c) 36
(d) 18	(e) 24	

38) Consider the following Venn diagram.



The shaded area corresponds to the event

(a) B^c	(b) $(A \cup B^c)^c$	(c) $A \cap B^c$
(d) $A^c \cap B$	(e) $A^c \cup B$	

39) Let A and B be mutually exclusive events with $P(A^c \cap B^c) = \frac{3}{8}$ and $P(A^c) = \frac{5}{8}$ Then P(B) =

(a) $\frac{3}{8}$	(b) $\frac{1}{4}$	(c) $\frac{1}{8}$
(d) $\frac{5}{8}$	(e) 0	

40) Suppose a fair die is tossed once.
Let A = {odd number occurs} and B = {prime number occurs}.
Then $P(A \cup B)$ =

(a) $\frac{2}{3}$	(b) $\frac{1}{3}$	(c) $\frac{1}{6}$
(d) $\frac{5}{6}$	(e) $\frac{4}{6}$	

41) Three players A, B and C are in a race. A is twice as likely to win as B and B is twice as likely to win as C. Then the probability that B wins =

(a) $\frac{1}{2}$	(b) $\frac{2}{3}$	(c) $\frac{1}{3}$
(d) $\frac{2}{7}$	(e) $\frac{5}{7}$	

42) A fair coin is tossed 3 times. Given that the first toss resulted in Heads, the probability that the other two were also Heads =

(a) $\frac{1}{2}$	(b) $\frac{1}{8}$	(c) $\frac{1}{4}$
(d) $\frac{3}{8}$	(e) $\frac{5}{8}$	

43) A company has 3 telephone lines, the manager's, the accountant's and the receptionist's.
At any given time on a normal day, the manager's phone is busy 80% of the time, the accountant's 50% of the time, and the receptionist's 90% of the time. If a customer picks one of their numbers at random and dials, the probability that his call is immediately connected is

(a) $\frac{4}{15}$	(b) $\frac{1}{15}$	(c) $\frac{4}{5}$
(d) $\frac{1}{5}$	(e) $\frac{3}{5}$	

44) Let the 6-tuple <B, +, *, ′, 0,1> be a Boolean algebra, where B be a set, + and * the sum and the product operators respectively and 0 and 1 the zero and the unit elements respectively and ′ the complement operator.
If a,b,c are elements of the set B which of the following is/are correct?

(a) a + b = b + a	(b) a * a′ = 1
(c) a + a′ = 0	(d) a + (b * c) = (a * b) + (a * c)
(e) a + (b * c) = (a + b) * (a + c)	

45) Let the 6-tuple <B, +, *, ′ , 0,1> be a Boolean algebra, where B be the set of n-bit sequences, and let + ,* , ′ operators be defined on each of the corresponding bits in any two bit sequences as below.

+	1	0
1	1	1
0	1	0

*	1	0
1	1	0
0	0	0

1′= 0
0′ = 1

If a = 101 and b = 010 which of the following is/are correct?

(a) a + b = 000	(b) a * b = 111
(c) a + b = 111	(d) a′ = 010
(e) a * b = 000	

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