UNIVERSITY OF COLOMBO, SRI LANKA

UNIVERSITY OF COLOMBO SCHOOL OF COMPUTING

DEGREE OF BACHELOR OF INFORMATION TECHNOLOGY (EXTERNAL)

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IT2105 - Mathematics for Computing I

26th July 2014

(TWO HOURS)

Important Instructions :

- The duration of the paper is 2 (two) hours.
- The medium of instruction and questions is English.
- The paper has **43** questions and **09** pages.
- All questions are of the MCQ (Multiple Choice Questions) type.
- All questions should be answered.
- Each question will have 5 (five) choices with **one or more** correct answers.
- All questions will carry equal marks.
- There will be a penalty for incorrect responses to discourage guessing.
- The mark given for a question will vary from 0 (*no correct choices are marked*) to +1 (*All the correct choices are marked & no incorrect choices are marked*).
- Answers should be marked on the special answer sheet provided.
- Note that questions appear on both sides of the paper.
- If a page is not printed, please inform the supervisor immediately.
- Mark the correct choices on the question paper first and then transfer them to the given answer sheet which will be machine marked. **Please completely read and follow the instructions given on the other side of the answer sheet before you shade your correct choices.**

Notations:

Z – set of integers N – set of positive integers

R – set of real numbers ∅ - (null) empty set

U – Universal set R^+ - set of positive real numbers

1) Which of the following is/are equal to $x^{\frac{2}{3}}$

(a) $\left(x^{\frac{1}{3}}\right)^2$.	(b) $\left(x^{\frac{1}{3}}\right)^{\frac{1}{2}}$.	(c) $\left(x^{\frac{1}{2}}\right)^{\frac{1}{3}}$.	(d) $\sqrt[3]{x^2}$.	(e) $\left(x^{\frac{4}{3}}\right)^{\frac{1}{2}}$.

2) $\frac{x^2yz^3}{x^3y^2z}$ is equal to

(a) $x^{-1}y^{-1}z^2$	(b) xyz^{-2}	(c) $x^5y^3z^4$	(d) $\frac{xy}{z^2}$	(e) $\frac{z^2}{xy}$

3) $\log_4 8$ is equal to

(a) 1.5 .	(b) $\log_2 4 - \log_2 8$.	(c) $\frac{\log_2 4}{\log_2 8}$.	(d) $\frac{2}{3}$.	(e) $\frac{\log_2 8}{\log_2 4}$.

4) Which of the following is(are) correct?

(a) $\forall a, u, v \in N \text{ and } a \neq 1, \log_a uv = \log_a u + \log_a v$.
(b) $\forall a, u, v \in N \text{ and } a \neq 1 \; \log_a uv = \log_a u - \log_a v$.
(c) $\forall a \in \{N \backslash 1\}, \log_a 1 = 0$.
(d) $\forall a \in \{N \backslash 1\}, \log_a 1 = 1$.
(e) $\forall a, u, v \in N \text{ and } a \neq 1 \; \log_a uv = (\log_a u)\,(\log_a v)$.

5) Let A = {x | x ∈ R and x^2-3x+2=0 } and B = {x | x ∈ R and x^2-5x+6=0 }.

A ∩ B is equal to

(a) {1}.	(b) {2}.	(c) {3}.	(d) {1,2}.	(e) {1,2,3} .

6) Let A = {0} ∪ N and B = {0} ∪ {-n | n ∈ N}.

A ∪ B is equal to

(a) N.	(b) Z.	(c) A.	(d) B.	(e) N ∪ {-n | n ∈ N}.

7) The sets A and B are such that $A \setminus B = \varnothing$. Which of the following is/are possible?

(a) $A \subseteq B$.	(b) $A \neq B$.	(c) $A = B$.	(d) $B \subseteq A$.	(e) $A = \varnothing$.

8) Let A and B be two non-empty **disjoint** sets. Which of the following is/are true?

(a) $A^c \cup B^c = (A \setminus B)^c \cap A$. (b) $A^c \cup B^c = (A \cap B)^c$. (c) $A^c \cup B^c = (A \setminus B)^c \cup A$
(d) $A^c \cup B^c = (A \setminus B)^c \cup B$. (e) $A^c \cup B^c = (B \setminus A)^c \cup B$.

9) Let A and B be any two non-empty sets. If A is a proper subset of B, which of the following(s) **must** be true?

(a) $A \subseteq B$.	(b) $A \neq B$	(c) $B \subset A$.
(d) $A \cap B \neq B$.	(e) $A \cap B \neq \varnothing$.	

10) Let A,B and C be three non-empty sets. Which of the following is(are) correct?

(a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$. (b) $A \cup (B \cap C) = (A \cap B) \cup (A \cap C)$.
(c) $A \cup (B \cap C) = (B \cup A) \cap (C \cup A)$. (d) $A \cup (B \cap C) = (B \cap A) \cup (C \cap A)$.
(e) $A \cup (B \cap C) = A \cap (B \cup C)$.

11) Let A and B be two sets. Which of the following is(are) correct?

(a) $A \cup B = \{x \mid x \in A \wedge x \in B\}$. (b) $A \cup B = \{x \mid x \in A \vee x \in B\}$.
(c) $A \cap B = \{x \mid x \in A \wedge x \in B\}$. (d) $A \cap B = \{x \mid x \in A \vee x \in B\}$.
(e) $(A \cup B)^c = \{x \mid x \notin A \wedge x \notin B\}$.

12) $(A \cap C) \cap B)$ is equal to

(a) $(A \cap C) \setminus B^c$	(b) $(A \cap B) \setminus C^c$	(c) $(B \cap C) \setminus A^c$
(d) $(A \cap C) \setminus A$	(e) $(A \cap B) \setminus A$	

13) Let A={ a, b, c, d } and B={ b, e }. Which of the following is/are propositions?

(a) $a \in B$.	(b) $A \cap B = \varnothing$.	(c) A is not a set.
(d) Is B a sub set of A?	(e) Find the complement of A.	

14) Which of the following propositions are/is logically equivalent to $\sim(p \leftrightarrow q)$?

(a) $(p \rightarrow q) \wedge (q \rightarrow p)$.	(b) $(p \wedge \sim q) \vee (\sim p \wedge \sim q)$.
(c) $(p \rightarrow q) \vee (q \rightarrow p)$.	(d) $\sim(p \rightarrow q) \wedge \sim(q \rightarrow p)$.
(e) $\sim(p \rightarrow q) \vee \sim(q \rightarrow p)$.	

15) Let *p* and *q* be two propositions. Which of the following are tautologies ?

(a) $(\sim p \vee q) \leftrightarrow \sim(p \wedge \sim q)$. _	(b) $(p \rightarrow q) \leftrightarrow \sim(p \wedge q)$.
(c) $(\sim p \vee q) \leftrightarrow (p \rightarrow q)$.	(d) $(\sim p \wedge q) \leftrightarrow \sim(p \wedge \sim q)$.
(e) $(p \rightarrow q) \leftrightarrow (\sim p \wedge q)$	

(16) Which of the following arguments is/are valid?

(a) $p \vee q, \sim p \vdash q$	(b) $p \vee q, \sim p \vdash \sim q$	(c) $p \Rightarrow q, \sim q \vdash p$
(d) $\sim p \vee q, p \vdash \sim q$	(e) $p \Rightarrow q, p \vdash q$	

(17) Which set(s) of the following statements is/are consistent?

(a) $p \wedge q, p \vee q, \sim p$	(b) $p \vee q, \sim p, q$	(c) $\sim(q \rightarrow p), q, \sim p$
(d) $\sim(q \rightarrow p), q, p$	(e) $\sim(q \rightarrow p), \sim q, \sim p$	

18) Consider the following truth tables for two different non-equivalent propositions of one variable ***p***.

p	**P1**	**P2**
T	**T**	**F**
F	**T**	**F**

Find two such propositions **P1, P2** in the given order.

(a) $p \vee \sim p, \sim(p \vee \sim p)$	(b) $p \vee \sim p, p \wedge p$
(c) $p \vee p, p \wedge p$	(d) $p \vee \sim p, p \wedge \sim p$
(e) $\sim(p \vee \sim p), p \wedge \sim p$	

19) Let $D=\{x_1, x_2, x_3,\ldots\ldots, x_n\}$ and the predicate $p(x)$ is defined on D. If $\forall x\, p(x)$ is true, which of the following are/is true?

(a) $p(x_1) \wedge p(x_2)$ must be false .	(b) $\exists x \sim p(x)$ must be false
(c) $p(x_1) \vee p(x_2)$ must be false.	(d) $\exists x\, p(x)$ must be true
(e) $p(x_1) \wedge p(x_2)$ must be true.	

20) Let $p(x)$ be a predicate defined on a domain D. If $\forall x\, p(x)$ is false, which of the following **MUST** be true?

(a) There is x_0 in D for which $p(x)$ is false.
(b) For every x in D, $p(x)$ is false.
(c) For every x in D, $\sim p(x)$ is true.
(d) There are no elements in D for which $p(x)$ is true.
(e) $\exists x\, p(x)$ is false.

21) Let $p(x)$: $x < 2$ and $q(x)$: $x \geq 2$ be two predicates of the variable x defined on N.

Which of the following propositions is(are) true?

(a) $\forall x\, (p(x) \wedge q(x))$	(b) $\forall x\, (p(x) \vee q(x))$	(c) $\forall x\, p(x)$
(d) $\exists x\, (p(x) \vee q(x))$	(e) $\exists x\, (p(x) \wedge q(x))$	

22) Suppose $x \in \{10, 20, 30, 40\}$ and $y \in \{6, 12, 16, 24, 25\}$. Which of the following propositions is(are) true?

(a) $\forall x\, \exists y\;\; x < y$.	(b) $\forall y\, \exists x\;\; x < y$.	(c) $\exists x\, \forall y\;\; x < y$.
(d) $\exists x\, \exists y\;\; x < y$.	(e) $\forall x\, \forall y\;\; x < y$.	

23) Let $p(x)$ be a predicate defined on a domain D. Which of the following is/are equivalent to $\sim\exists x\, p(x)$?

(a) $\forall x\, p(x)$.
(b) $\exists x \sim p(x)$.
(c) $\forall x \sim p(x)$.
(d) $\sim\forall x \sim p(x)$.
(e) $\sim\exists x \sim p(x)$.

24) Let $X = \{3, 4, 6\}$, $Y=\{1, 2, 8, 9\}$, $\alpha=\{(a,b) \mid a\in X, b\in Y, a > b\}$.

Which of the following belong to α?

(a) (6,3).	(b) (6,2).	(c) (6,9).
(d) (2,1).	(e) (7,3).	

25) Let α and β be two relations defined by
$\alpha = \{(x,y) \mid x\in Z, y\in Z, x \leq y\}$ and $\beta = \{(x,y) \mid x\in Z, y\in Z, x > y\}$.

Which of the following is/are true?

(a) α and β are not symmetric.
(b) α and β are reflexive.
(c) α is symmetric and β is not symmetric.
(d) α is reflexive and β is not reflexive.
(e) α is transitive and β is not transitive.

26) Let α be a relation defined on Z by $\alpha = \{(x,y) \mid x\in Z, y\in Z, x \leq y\}$.

What is α^{-1}?

(a) $\alpha^{-1} = \{(x,y) \mid x\in Z, y\in Z, y > x\}$.
(b) $\alpha^{-1} = \{(x,y) \mid (y,x)\in \alpha\}$.
(c) $\alpha^{-1} = \{(x,y) \mid x\in Z, y\in Z, x > y\}$.
(d) $\alpha^{-1} = \{(x,y) \mid x\in Z, y\in Z, \sim(x \leq y)\}$.
(e) $\alpha^{-1} = \{(x,y) \mid x\in Z, y\in Z, x \geq y\}$.

27) Let α be a relation defined on a non-empty set D by $\alpha = \{(x,y) \mid x,y\in D\}$. Then α is said to be symmetric if and only if

(a) $\forall x\ (x,x)\in\alpha$.
(b) $\forall x\forall y\forall z\ (x,y)\in\alpha \wedge (z,y)\in\alpha \rightarrow (x,z)\in\alpha$.
(c) $\forall x\forall y\ (x,y)\in\alpha \rightarrow (y,x) \in \alpha$.
(d) $\exists x,\ x \in D(\rho) \wedge (x,x) \in \rho$.
(e) $\forall x\forall y\ (x,y)\notin\alpha \vee (y,x)\in\alpha$.

28) Suppose $A=\{10,15,20\}$.
If $\alpha = \{(x,y) \mid x,y \in A,\ x < y\}$ and $\beta = \{(x,y) \mid x,y \in A,\ n\in N,\ x = ny\}$,
which of the following is/are true?

(a) $\alpha\mathbf{o}\beta = \{(10,10), (10,15),(10,20),(15,10),(15,20)\}$.
(b) $\alpha\mathbf{o}\beta = \alpha$.
(c) $\alpha\mathbf{o}\beta = \{(10,15),(10,20),(15,20),(20,15), (20,20)\}$.
(d) $\alpha\mathbf{o}\beta = \beta\mathbf{o}\alpha$.
(e) $\alpha\mathbf{o}\beta = \beta$.

29) Let ρ be the relation defined on A={a,b,c} by

ρ={(a,a),(b,b),(c,c),(a,b),(b,a),(b,c),(c,b),(a,c),(c,a)}.

Find $[a]_\rho \cap [b]_\rho$.

(a) {a,b}.
(b) {a}.
(c) $\varnothing$.
(d) {c}.
(e) A.

30) Which of the following is not a proper representation of a function f on A={1,2,3}?

(a) $f(1)=10, f(2)=10, f(3)=10$.
(b) $f(1)=8, f(2)=8, f(3)=10$.
(c) $f(1)=7, f(1)=8, f(2)=9, f(3)=10$.
(d) $f(1)=8, f(2)=9$.
(e) $f(1)=1, f(2)=2, f(3)=3$.

31) Suppose f is a 1-1 function and $x,y \in D(f)$. Which of the following are(is) correct about f?

(a) $\forall x \forall y \;\; x \neq y \Rightarrow f(x) = f(y)$.
(b) $\forall x \forall y \;\; \sim(x = y) \Rightarrow \sim(f(x) = f(y))$.
(c) $\forall x \forall y \;\; f(x) = f(y) \Rightarrow x = y$.
(d) $\forall x \forall y \;\; \sim(f(x) = f(y)) \Rightarrow x \neq y$.
(e) $\sim (\exists x \, \exists y \sim(x = y) \wedge \; f(x) = \; f(y))$.

32) Let the functions f and g be defined by $f(x) = 2x-1$ and $g(x) = 3x$ where $x \in R$.
Then $(f \circ g)(x)$ is equal to

(a) 6x-3.
(b) 6x-1.
(c) 6x.
(d) 3x-2.
(e) 2x-3.

33) Let A={1,2,3} and B={1,4,9}and f and g be bijections from A to B. Which of the following functions are/is bijections from A to B?

(a) $\forall x \in A \; h(x)=x^2+5$.
(b) $g^{-1} \circ f^{-1}$.
(c) f^{-1}.
(d) $g \circ f^{-1}$.
(e) $\forall x \in A \; h(x)=x^2$.

34) Let the 6-tuple <B,+,*, c,0,1> be a Boolean algebra where B is a set, + and * the sum and the product operators respectively, 0 and 1 the zero and the unit elements respectively and c the complement operator.

If b is an element of the set B, what is the dual of the Boolean expression b + 1 = 1?

(a) b * 1 = 1.	(b) b * 0 = 0.	(c) b + 0 = 0.
(d) b * A = A.	(e) b * 1 = 1.	

35) Find the number of arrangements that can be made by taking all the letters in the word "PROPOSITION" if the three letters "O" are together?

(a) $\frac{11!}{(2!)(2!)(3!)}$	(b) $\frac{9!}{(2!)(2!)}$	(c) $\frac{9!}{4}$	(d) $\frac{11!}{4!}$	(e) $(2!)(2!)(3!)$

36) There are 6 boys and 4 girls. Find the number of ways in which they can be arranged in a line such that no two girls are together.

(a) $\frac{7!}{4!}$	(b) 15	(c) $\frac{6!}{(4!)(2!)}$	(d) $\frac{7!}{(4!)(3!)}$	(e) 35

37) A coin is tossed three times in a succession, and the total number of times heads comes up is noted. The sample space is,

(a) {0, 1, 2, 3}.
(b) {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}.
(c) {HHH, HHT, HTH, HTT, THH, THT, TTH}.
(d) {H, T}.
(e) {1,2,3}.

38) In an experiment, a pair of dice (one red, one green) is thrown and the number facing up on each die is noted. Let E be the event that the sum of the numbers is 4, and F be the event that the sum is an odd number. $E' \cup F$ is the event

(a) that the sum of the numbers is an odd number.
(b) that the sum of the numbers is any number.
(c) that the sum of the numbers is any even number other than 4.
(d) that the sum of the numbers is any number other than 4.
(e) which is the null set.,

39) The probability that an American industry will locate in Shanghai, China is 0.7, the probability that it will locate in Beijing, China is 0.4, and the probability that it will locate in either Shanghai or Beijing or both is 0.8. What is the probability that the industry will locate in neither city?

(a) 0.8	(b) 0.3	(c) 0.2
(d) 0.7	(e) Provided information is not sufficient	

40) In the following functions, which constitute S={a,b,c} as a probability space:

(a) P(a) = 0.5 P(b) = 0.3 P(c) = 0.3
(b) P(a) = 0.5 P(b) = 0.2 P(c) = 0.3
(c) P(a) = 0.5 P(b) = -0.3 P(c) = 0.3
(d) P(a) = 0.5 P(b) = 0.7 P(c) = - 0.2
(e) P(a) = 0.5 P(b) = 0.7 P(c) = 0.2

(41) The probability that a married man watches a certain television show is 0.4 and the probability that a married woman watches the show is 0.5. The probability that a married man watches the show, given that his wife does is, 0.7. What is the probability that the married couple watches the show?

(a) 0.7	(b) 0.2	(c) 0.35
(d) 0.9	(e) 0.28	

42) Bag A contains 10 marbles of which 2 are red and 8 are black. Bag B contains 12 marbles of which 4 are red and 8 are black. A ball is drawn at random from each bag. What is the probability that at least one is red?

(a) $\frac{8}{15}$	(b) $\frac{7}{15}$	(c) $\frac{1}{2}$
(d) $\frac{1}{15}$	(e) $\frac{6}{22}$	

43) If A and B are independent events, what is P(A|B)?

(a) P(B|A).	(b) P(A).	(c) P(B).
(d) P(A∪B).	(e) P(A ∩ B).	

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