UNIVERSITY OF COLOMBO, SRI LANKA

UNIVERSITY OF COLOMBO SCHOOL OF COMPUTING

DEGREE OF BACHELOR OF INFORMATION TECHNOLOGY (EXTERNAL)

Academic Year 2010 /2011 – 1st Year Examination – Semester 2

IT2104 - Mathematics for Computing I

30th July 2011

(TWO HOURS)

Important Instructions :

- The duration of the paper is 2 (two) hours.
- The medium of instruction and questions is English.
- The paper has **45** questions and **10** pages.
- All questions are of the MCQ (Multiple Choice Questions) type.
- All questions should be answered.
- Each question will have 5 (five) choices with **one or more** correct answers.
- All questions will carry equal marks.
- There will be a penalty for incorrect responses to discourage guessing.
- The mark given for a question will vary from 0 (*no correct choices are marked*) to +1 (*All the correct choices are marked & no incorrect choices are marked*).
- Answers should be marked on the special answer sheet provided.
- Note that questions appear on both sides of the paper.
- If a page is not printed, please inform the supervisor immediately.
- Mark the correct choices on the question paper first and then transfer them to the given answer sheet which will be machine marked. **Please completely read and follow the instructions given on the other side of the answer sheet before you shade your correct choices.**

Notations:

Z – set of integers N – set of positive integers
R – set of real numbers ∅ - (null) empty set
U – Universal set R^+ - set of positive real numbers

1) $\frac{16^{1/2} \times 2^2}{16^{3/2} \times 2^0}$ is equal to

(a) $\frac{2^2}{16 \times 2^0}$ (b) $\frac{1}{4}$ (c) $\frac{(4^2)^{1/2} \times 2^2}{(4^2)^{3/2} \times 2^0}$ (d) $\frac{1}{2}$ (e) $\frac{1}{16}$

2) Which of the following is(are) correct?

(a) $\forall a \in N, \forall u, v \in R^+ \ \log_a(uv) = \log_a u + \log_a v$
(b) $\forall a \in N, \forall u, v \in R \ \log_a(uv) = \log_a u + \log_a v$
(c) $\forall a \in N, \log_a 1 = 0$
(d) $\forall a \in Z, \log_a 1 = 0$
(e) $\forall a \in N, \log_a a = 1$

3) $\log_{10} 54 + \log_{10} 5 - 3\log_{10} 3$ is equal to

(a) $\log_{10} 90$ (b) $\log_{10} 30$ (c) $\log_{10} 10$ (d) $\log_{10} 1$ (e) 1

4) Let S = {(x,y)| x,y □ Z and $x^2 + y^2 = 17$} and T = {(x,y)| x,y □ Z and x - y = 5}. Then S ∩ T equals

(a) {(4,1)} (b) {(1,4)} (c) {(4,-1)} (d) {(1,4), (4,-1)} (e) {(4,-1),(1,-4)}

5) Let A and B be two non-empty sets. Which of the following is/are true?

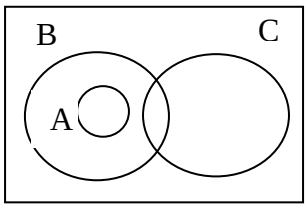
(a) A\B ⊆ A (b) A\B ⊆ B (c) (A\B) ∩ B = ∅ (d)(A ∩ (A\B) ∩ B^c = ∅
(e) A ∩ (A\B) ∩ B^c =A\B

6) Let A and B be any two non-empty sets. If A is not a proper subset of B, which of the following could be true?

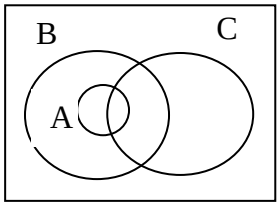
(a) $A = B$. (b) $A \neq B$. (c) $B \subset A$. (d) $A \subseteq B$ and $A \neq B$. (e) $A \cap B = \varnothing$.

7) Let A,B and C be three non-empty sets such that $A \subseteq B$, $C \not\subseteq B$ and $A \cap C \neq \varnothing$. Which of the following Venn diagrams reflect these properties?

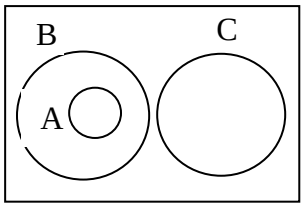
(a)



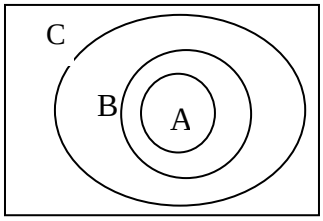
(b)



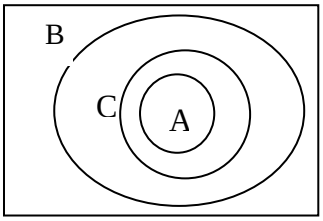
(c)



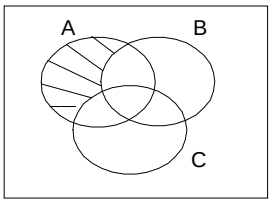
(d)



(e)



8) Consider the following Venn diagram.



Which of the following sets is represented by the shaded portions?

(a) $(A \cap B)^c$. (b) $A^c \cap (B \cup A)$.
(c) $(A \cup B) \cap (A \cup C)$. (d) $(B \cup C)^c \cap A$.
(e) $(A \cap C)^c$.

9) Let A be a non empty set. Which of the following is(are) correct?

(a) $A \subset A$ (b) $\varnothing \subset A$
(c) $A \subseteq U$ (d) $A \subseteq A$
(e) $\varnothing \subseteq A$

10) Let A and B be any two sets. Which of the following are propositions?

(a) The word "queue" has three vowels.
(b) $A \subset A$.
(c) $A \subseteq B$.
(d) $A \cap A^c = \varnothing$.
(e) Write your index number on the answer script.

11) Let *p* and *q* be two atomic propositions. Which of the following is(are) a tautologies involving p and q?

(a) $p \wedge q \Rightarrow p \vee q$.
(b) $p \vee \sim q$.
(c) $p \vee (q \vee \sim q)$.
(d) $p \wedge (q \wedge \sim q)$.
(e) $(p \wedge q \Rightarrow p \vee q) \vee \sim q$.

12) Consider the following truth table of the proposition Ω with three propositional variables *p*, *q* and *r*.

p	*q*	*r*	Ω
T	T	T	T
T	T	F	F
T	F	T	T
F	T	T	F
T	F	F	T
F	T	F	T
F	F	T	F
F	F	F	T

Which of the following could be Ω?

(a) $(p \wedge q \Rightarrow p \vee q) \vee \sim r$
(b) $(p \wedge q \wedge r) \vee (p \wedge \sim q \wedge r) \vee (p \wedge \sim q \wedge \sim r) \vee (\sim p \wedge q \wedge \sim r) \vee (\sim p \wedge \sim q \wedge \sim r)$
(c) $(p \wedge q \wedge \sim r) \vee (\sim p \wedge q \wedge r) \vee (\sim p \wedge \sim q \wedge r)$
(d) $(p \vee q \vee r) \wedge (p \vee \sim q \vee r) \wedge (p \vee \sim q \vee \sim r) \wedge (\sim p \vee q \vee \sim r) \wedge (\sim p \vee \sim q \vee \sim r)$
(e) $(\sim p \vee \sim q \vee r) \wedge (p \vee \sim q \vee \sim r) \wedge (\sim p \vee \sim q \vee r)$

13) Which of the following pairs of propositions is(are) equivalent?

(a) $(p \Rightarrow q)$, $p \vee \sim q$
(b) $p \wedge (\sim p \vee q)$, $p \wedge q$
(c) $p \wedge (\sim p \vee q)$, q
(d) $\sim p \vee (q \wedge r)$,$(\sim p \vee q) \wedge (\sim p \vee r)$
(e) $(p \Rightarrow q)$, $(\sim q \Rightarrow \sim p)$

14) Suppose that when you left the home, you found that your mobile phone is not with you. You know that the following statements are true.

i) I was reading the newspaper in the living room or in the kitchen.
ii) If I was reading the newspaper in the living room, my mobile phone is on the coffee table.
iii) mobile phone is not in the coffee table.
iv) If I was reading the newspaper in the kitchen, my mobile phone is on the kitchen table.

Which of the following is(are) true?

(a) The mobile phone is on the kitchen table.
(b) The mobile phone is not on the kitchen table.
(c) I was reading the newspaper in the kitchen.
(d) I was reading the newspaper in the living room.
(e) I was not reading the newspaper in the living room.

15) Which set(s) of the following statements is(are) consistent?

(a) $p \wedge q$, p, q (b) $p \vee q$, $\sim p$, $\sim q$ (c) $(q \Rightarrow p)$, $(p \Rightarrow \sim r)$, $\sim q$, r
(d) $(q \Leftrightarrow p)$, $\sim p$, q (e) $\sim q \Rightarrow p$, $p \Rightarrow \sim r$, r

16) Which of the following arguments is(are) valid?

(a) $p, (p \Rightarrow q) \vdash q$ (b) $(p \Rightarrow q), q \vdash p$ (c) $(p \Rightarrow q), (p \vee r), (r \Rightarrow \sim s), s \vdash q$
(d) $p, (\sim p \vee q) \vdash q$ (e) $(p \Rightarrow q), \sim q \vdash \sim p$

17) Let p(x) and q(x) be two predicates of the variable x defined by $x \leq 2$ and $x > 2$ respectively where $x \in R$.

Which of the following propositions is(are) true?

(a) $\exists x\, p(x) \vee \exists x\, q(x)$ (b) $\exists x\, (p(x) \vee q(x))$ (c) $\forall x\, (p(x) \vee q(x))$
(d) $\forall x\, p(x) \vee \forall x\, q(x)$ (e) $\forall x\, (p(x) \wedge q(x))$

18) Consider the following propositions.

(i) $\forall x\, p(x)$ (ii) $\exists x\, p(x)$ (iii) $\forall x \sim p(x)$ (iv) $\sim \exists x\, p(x)$ (v) $\sim \forall x \sim p(x)$ (vi) $\sim \exists x \sim p(x)$

Identify the equivalent propositions to the above.

(a) (i) and (vi) (b) (i) and (iv) (c) (ii) and (v)
(d) (iii) and (iv) (e) (ii) and (iii)

19) Suppose $x \in \{7, 18, 24, 31\}$ and $y \in \{5, 10, 15, 20, 25, 30, 35\}$.

Which of the following propositions is(are) true?

(a) $\forall x \exists y\ x < y$.	(b) $\forall y \exists x\ y < x$.	(c) $\exists x \forall y\ x < y$.
(d) $\exists y \forall x\ y < x$.	(e) $\forall x \forall y\ x < y$.	

20) Let p and q be two atomic propositions. Which of the following propositions is(are) expressed in Disjunctive Normal Form?

(a) $(p \vee q \vee r) \wedge (p \vee \sim q \vee r) \wedge (\sim p \vee q \vee r)$.	(b) $(p \vee q) \wedge (p \vee \sim q \vee r) \wedge (\sim r \vee s)$.
(c) $p \vee q$.	(d) $(p \wedge q \wedge r) \vee (p \wedge \sim q \wedge r) \vee (\sim p \wedge q \wedge r)$.
(e) p.	

21) Which of the following is(are) true?

(a) A valid argument with one or more false premises may have a true conclusion.
(b) A valid argument with one or more false premises may have a false conclusion.
(c) A valid argument with all premises true may have false conclusion.
(d) An invalid argument(fallacy) with all premises true may have a true conclusion.
(e) An invalid argument(fallacy) with all premises true may have a false conclusion.

22) Let T = {1,2,3} and p(x) is a predicate of variable x defined on T.

Which of the following propositions is(are) true if $\exists x\ p(x)$ is false ?

(a) $p(1) \vee p(2) \vee p(3)$	(b) $p(1) \wedge p(2) \wedge p(3)$	(c) $\sim p(1) \vee \sim p(2) \vee \sim p(3)$
(d) $\sim p(1) \wedge \sim p(2) \wedge \sim p(3)$	(e) $\forall x \sim p(x)$	

23) Let A = {1, 2} and B={3,4}. The Cartesian Product, A x B is equal to?

(a) {(1,3), (2,4)}.	(b) {(3,1), (4,2)}.	(c) {(1,3), (1,4),(2,3),(2,4)}.
(d) {(3,1), (3,2),(4,1),(4,2)}.	(e) {(1,2),(3,4)}.	

24) Suppose a relation ρ is non-empty and defined on a set X. Then ρ is said to be symmetric if

(a) $\forall x,\ x \in D(\rho) \Rightarrow (x,x) \in \rho$	(b) $\forall x \forall y,\ (x,y) \in \rho \Rightarrow (y,x) \in \rho$
(c) $\forall x \forall y,\ (x,y) \in \rho \wedge (y,x) \in \rho$	(d) $\forall x \forall y \forall z,\ (x,y) \in \rho \wedge (y,z) \in \rho \Rightarrow (x,z) \in \rho$
(e) $\forall x \forall y \forall z,\ (x,y) \in \rho \wedge (y,z) \in \rho \wedge (x,z) \in \rho$	

25) Let A={4,6} and B={3,12,18} and β={ (x,y)| x$\in$ A, y$\in$ B, x divides y}.

Which of the following is(are) true?

(a) β^{-1} ={(x,y)| x$\in$ B, y$\in$ A, y divides x }.	(b) β^{-1} ={(4,12),(4,18),(6,12),(6,18)}.
(c) β^{-1} ={(12,4),(12,6),(18,6)}.	(d) β^{-1} ={(y,x)| (x,y) $\in\beta$}.
(e) β^{-1} ={(3,4)}.	

26) Let α and ρ be two relations given by
α={(5,6),(7,9),(8,3),(4,4)} and ρ={(6,1),(9,9),(8,5),(6,12),(10,4)}.

Then $\rho \mathbf{o} \alpha$ equals

(a) {(5,1),(5,12),(7,9)}
(b) {(8,6),(10,4)}
(c) {(5,6),(7,9),(8,3),(4,4),(6,1),(9,9),(8,5),(6,12),(10,4)}
(d) {(5,1),(5,12)}
(e) {(8,6)}

27) ρ = { (1,1), (2,2), (3,3), (4,4), (5,5), (6,6), (1,2), (2,1), (2,3), (3,2), (1,3), (3,1), (4,5), (5,4) } is an equivalence relation.

$[1]_\rho \cup [2]_\rho \cup [3]_\rho \cup [4]_\rho \cup [5]_\rho \cup [6]_\rho$ equals

(a) {1,2,3}.	(b) {4,5}.	(c) {6}.
(d) {1,2,3,4,5,6}.	(e) D(ρ).	

28) Let A be a non-empty sub set of N.
Suppose α={(x,y)| x,y $\in$ A, x divides y} and β={(x,y)| x,y$\in$ A, x < y}.

Which of the following is(are) true?

(a) α is Reflexive and Symmetric.	(b) α is Reflexive and Transitive.
(c) β is Reflexive and Symmetric.	(d) β is Reflexive and Transitive.
(e) α and β are both Transitive.	

29) Suppose ρ is a relation. Which of the following is(are) true?

(a) D(ρ)={ y | $\exists$x (x,y)$\in$ ρ},	R(ρ)={ x | $\exists$y (x,y)$\in$ ρ}.
(b) D(ρ^{-1})={ y | $\exists$x (x,y)$\in$ ρ},	R(ρ^{-1})={ x | $\exists$y (x,y)$\in$ ρ}.
(c) D(ρ)={ x | $\exists$y (x,y)$\in$ ρ},	R(ρ)={ y | $\exists$x (x,y)$\in$ ρ}.
(d) D(ρ^{-1})={ x | $\exists$y (x,y)$\in$ ρ},	R(ρ^{-1})={ y | $\exists$x (x,y)$\in$ ρ}.
(e) D($\rho \mathbf{o} \rho$)=R(ρ).	

30) Which of the following is(are) true if f is a one to one function?

(a) $\forall x_1, \forall x_2 \; x_1 \in D(f), x_2 \in D(f), \; x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$
(b) $\forall x_1, \forall x_2 \; x_1 \in D(f), x_2 \in D(f), \; x_1 = x_2 \Rightarrow f(x_1) \neq f(x_2)$
(c) $\forall x_1, \forall x_2 , \; x_1 \in D(f), x_2 \in D(f), \; f(x_1) = f(x_2) \Rightarrow x_1 = x_2$
(d) $\forall x_1, \forall x_2 , \; x_1 \in D(f), x_2 \in D(f), \; f(x_1) = f(x_2) \Rightarrow x_1 \neq x_2$
(e) $\forall x_1, \forall x_2 , x_1 \in D(f), x_2 \in D(f), \; f(x_1) \neq f(x_2) \Rightarrow x_1 = x_2$

31) Let B be a non-empty set. Which of the following is(are) true if the functions f and g are bijections from B onto B?

(a) $f \, o \, g$ is a bijection from B onto B
(b) $D(f) = B, \; R(f) \subset B$
(c) f^{-1} is a bijection from B onto B
(d) $D(f) \subset B, \; R(f) = B$.
(e) $f \, o \, f^{-1}$ is a bijection from B onto B .

32) Which of the following functions is(are) one to one?

(a) $D(f)=R$ and $f(x)=2x+1$ for $x \in R$
(b) $D(f)=R$ and $f(x)= x^2+1$ for $x \in R$
(c) $D(f)=\{x| \; x \in R , x > 0\}$ and $f(x)= x^2+1$ for $x \in D(f)$
(d) $D(f)=R$ and $f(x) = |x|$ for $x \in R$
(e) $D(f)=\{x| \; x \in R , x > 0\}$ and $f(x)= |x|$ for $x \in D(f)$

33) Suppose A,B and C are three non-empty sets. If f maps A into B and g maps B onto C, which of the following is/are true?

(a) $D(f) = A, \; R(f) \subseteq B$.
(b) $D(g \, o \, f) = A, \; R(g \, o \, f) \subseteq C$
(c) $D(g \, o \, f) = A, \; R(g \, o \, f) = C$
(d) $D(g) = B, \; R(g) \subset C$.
(e) $D(g) = B, \; R(g) = C$.

34) Consider the function f defined by $f(x)=2x-1$ for $x \in R$. Which of the following is(are) true?

(a) $D(f^{-1})=R$ and $f^{-1}(x)=2x+1$ for $x \in R$
(b) $D(f^{-1})=R$ and $f^{-1}(x)= (x-1)/2$ for $x \in R$
(c) $D(f^{-1})=\{x| \; x \in R , x>0\}$ and $f^{-1}(x)= (x-1)/2$ for $x \in D(f^{-1})$
(d) $D(f^{-1})=R$ and $f^{-1}(x)= (x+1)/2$ for $x \in R$
(e) $D(f^{-1})=\{x| \; x \in R , x > 0\}$ and $f^{-1}(x)= (x+1)/2$ for $x \in D(f^{-1})$

35) How many permutations of 3 **different** digits are there, chosen from the ten digits 0 to 9 inclusive?

(a) 84
(b) 120
(c) 504
(d) 720
(e) 60.

36) In how many ways can a committee of 5 be chosen from 10 people?

(a) $^{10}C_5$.	(b) $^{10}P_5$.	(c) 252.
(d) 30,240.	(e) $^{10}C_5$*5!	

37) Nimal is the Chairman of a committee. In how many ways such a committee of 5 be chosen from 10 people?

(a) (9×8×7×6)/(4×3×2×1)	(b) 9×8×7×6	(c) 9×8×7×6×5
(d) (9×8×7×6×5)/(4×3×2×1)	(e) 126	

38) Let (B, $\vee$, $\wedge$) be a Boolean algebra with B representing set of propositions. If p and q are two propositions, find the dual of $(T \vee p) \wedge (q \vee F) \equiv q$

(a) $(T \wedge p) \vee (q \wedge F) \equiv q$	(b) $(F \vee p) \wedge (q \vee T) \equiv q$	(c) $(F \wedge p) \vee (q \wedge T) \equiv q$
(d) $(F \wedge q) \vee (p \wedge T) \equiv p$	(e) $(T \wedge q) \vee (p \wedge F) \equiv p$	

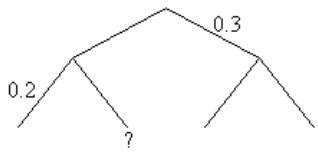
39) If $A \subset B$ then $P(A|B)$ is equals to

(a) 0	(b) 1	(c) $\frac{P(B)}{P(A)}$
(d) $\frac{P(A)}{P(B)}$	(e) $P(A)$	

40) A die is rolled and a coin is tossed. Find the probability that the die shows an odd number and the coin shows a head.

(a) 1/12	(b) 7/12	(c) 6/12	(d) 4/12	(e) 3/12

41) What is the missing joint probability value shown by a question mark on the tree diagram given below?



(a) 0.50	(b) 0.70	(c) 0.90	(d) 0.56	(e) 0.75

42) A box contains 3 red apples and 2 green apples. Three apples are drawn at random without replacement. What is the probability that two apples are red and one apple is green?

(a) 0.6	(b) 0.5	(c) 0.4	(d) 0.3	(e) 0.2

43) What is the probability that a card selected from a deck with 52 playing cards will be either an ace or a heart?

(a) 2/52	(b) 2/13	(c) 7/26	(d) 4/13	(e) 17/52

44) A statistics teacher gave his class two tests. 30% of the class passed both tests and 45% of the class passed the first test. What percent of those who passed the first test also passed the second test?

(a) 30%	(b) 75.0%	(c) 66.7%	(d) 150.0%	(e) 1.5%

45) An outdoor night party has been arranged for the day after the wedding by a newly wedded couple. In recent years, it has rained on only 5 days during this month (assume a 30 day month). Unfortunately, the Meteorological Department has predicted rain for that day. The Meteorological Department correctly forecasts that it will rain 75% of the time and incorrectly forecasts rain 25% of the time. What is the probability that it will rain on the day of the outdoor party, given a forecast of rain by the Meteorological Department?

(a) 0.125	(b) 0.375	(c) 0.625	(d) 0.938	(e) 0.333

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