UNIVERSITY OF COLOMBO, SRI LANKA

UNIVERSITY OF COLOMBO SCHOOL OF COMPUTING

DEGREE OF BACHELOR OF INFORMATION TECHNOLOGY
Academic Year 2009/2010 – 1st Year Examination – Semester 2

IT2104 - Mathematics for Computing I

07th August 2010
(TWO HOURS)

Important Instructions :

- The duration of the paper is **2 (two)** hours.
- The medium of instruction and questions is English.
- The paper has **43** questions and **10** pages.
- All questions are of the MCQ (Multiple Choice Questions) type.
- All questions should be answered.
- Each question will have 5 (five) choices with **one or more** correct answers.
- All questions will carry equal marks.
- There will be a penalty for incorrect responses to discourage guessing.
- The mark given for a question will vary from 0 to +1 (*All the correct choices are marked & no incorrect choices are marked*).
- Answers should be marked on the special answer sheet provided.
- Note that questions appear on both sides of the paper.
- If a page is not printed, please inform the supervisor immediately.
- Mark the correct choices on the question paper first and then transfer them to the given answer sheet which will be machine marked. **Please completely read and follow the instructions given on the other side of the answer sheet before you shade your correct choices.**

Notations:

Z – set of integers N – set of positive integers
R – set of real numbers ∅ - (null) empty set
S – Universal set

1) Evaluate $5(6x^3y^4z)^{1/4}$ when $x = 3, y = 1, z = 8$

(a) 180 (b) 30 (c) 90 (d) 6 (e) 36

2) $\dfrac{p^{\frac{5}{6}} \times q^{\frac{2}{3}}}{p^{\frac{1}{6}} \times q^{\frac{1}{2}}}$ is equal to

(a) $p^{\frac{4}{6}} \times q^{\frac{1}{3}}$ (b) $(p^2 \times q)^{1/3}$ (c) $p^{\frac{4}{6}} \times q^{\frac{1}{6}}$ (d) $(p^4 \times q)^{1/6}$ (e) $(p\,q)^{5/6}$

3) $(-3) + \log_2 24$ is equal to

(a) $\log_2 8$ (b) $\log_2 3$ (c) $(\log_3 2)^{-1}$ (d) $\log_2 10 * \log_{10} 3$
(e) $\log_{10} 2 * \log_3 10$

4) Let A = {1,4,8,9} and B= {3, 4, 6, 9, 10}. Consider the following statements.

(i) A ∪ B = {1,4,8,9,3,4,6,9,10}
(ii) A ∩ B = {4,9}
(iii) A \ B = {1,8}.

Then

(a) only (i) and (ii) are true.
(b) only (ii) is true.
(c) only (i) and (iii) are true.
(d) (i), (ii) and (iii) are all true.
(e) only (ii) and (iii) are true.

5) Let A and B be two non-empty sets. Which of the following is/are true?

(a) A ∩ (A ∪ B) = A.
(b) A ∩ (A ∪ B) = B.
(c) A \ (A ∩ B) = A.
(d) A ∪ (A ∩ B) = A.
(e) A ∪ (A ∩ B) = B.

6) Let A and B be any two non-empty sets. If A is not a proper subset of B, which of the following must be false?

(a) A ⊆ B.
(b) A ≠ B.
(c) B ⊂ A.
(d) (A ⊆ B) ∧ (A ≠ B).
(e) A ∩ B = ∅.

7) $S = \{(a, b) \mid a,b \in Z,\ 2a + b = 3\}$
$T = \{(a, b) \mid a, b \in Z,\ 4a + 3b = 8\}$

$S \cap T$ is equal to

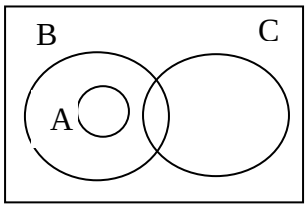
(a) $\{2,-2\}$.	(b) $\{-2,2\}$.	(c) $\{(1/2, 2)\}$.
(d) $\{(-2, 1/2)\}$.	(e) $\varnothing$.	

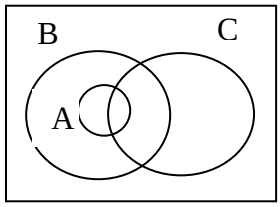
8) $A = \{-2, -1, 0\} \cup N$ and $B = \{2, 1, 0\} \cup \{-n \mid n \in N\}$.

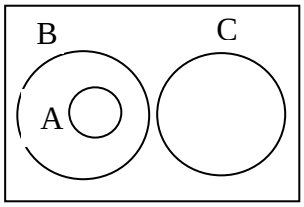
Which of the following is/are true?

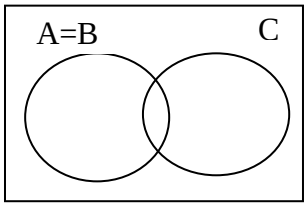
(a) $A \setminus B = \{n \mid n \in N \text{ and } n > 2\}$ and $B \setminus A = \{-n \mid n \in N \text{ and } n > 2\}$.
(b) $A \setminus B = \{n \mid n \in N \text{ and } n > 2\}$ and $B \setminus A = \{ n \mid n \in Z \text{ and } n < -2\}$.
(c) $A \setminus B = \{-2, -1, 0\}$ and $B \setminus A = \{-n \mid n \in N \text{ and } n > 2\}$.
(d) $A \setminus B = \{n \mid n \in N \text{ and } n > 2\}$ and $B \setminus A = \{2, 1, 0\}$.
(e) $A \setminus B = \{-2, -1, 0\}$ and $B \setminus A = \{2, 1, 0\}$.

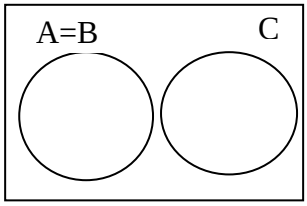
9) Let A,B and C be three non-empty sets such that $A \subseteq B$, $C \not\subset B$ and $A \cap C = \varnothing$. Which of the following Venn diagrams reflect these properties ?

(a)


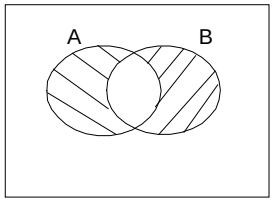
(b)


(c)


(d)


(e)


10) Consider the following Venn diagram.



The shaded portion in the Venn diagram represents

(a) $(A \cap B)^c$.	(b) $(A^c \cup B) \cap (B^c \cup A)$.
(c) $(A \cup B^c)^c \cup (B \cup A^c)^c$.	(d) $(A \cap B)^c \cap (A \cup B)$.
(e) $(A^c \cap B) \cup (B^c \cap A)$.	

11) $((A \cap C) \cup B) \cap ((A \cap B) \cup C)$ is equal to

(a) $\varnothing$.	(b) $A \cup C$.
(c) $A \cap B \cap C$.	(d) $(A \cap C) \cup (A \cap B) \cup (B \cap C)$.
(e) $((A \cup C) \cap B) \cup ((A \cap C) \cup B)$	

12) Consider the following truth table for four different non-equivalent propositions of one variable ***p***.

p	P1	P2	P3	P4
T	T	T	F	F
F	T	F	T	F

Find four such propositions P1, P2, P3, P4 in the given order.

(a) $p \vee \sim p,\ p,\ \sim p,\ \sim (p \vee \sim p)$	(b) $p \vee \sim p,\ p,\ \sim p,\ p \wedge p$
(c) $p \vee p,\ p,\ \sim p,\ p \wedge p$	(d) $p \vee \sim p,\ p,\ \sim p,\ p \wedge \sim p$
(e) $\sim(p \wedge \sim p),\ p,\ \sim p,\ p \wedge \sim p$	

13) Suppose when you left the house, you found that your mobile phone is not with you. You know the following statements are true.

i) If my mobile phone is on the kitchen table, I saw it at breakfast.
ii) I was reading the newspaper in the living room or in the kitchen.
iii) If I was reading the newspaper in the living room, my mobile phone is on the coffee table.
iv) I did not see my mobile phone at breakfast.
v) If I was reading the newspaper in the kitchen, my mobile phone is on the kitchen table.

Which of the following is/are true?

(a) The mobile phone is on the kitchen table.
(b) The mobile phone is not on the coffee table.
(c) I was not reading the newspaper in the kitchen.
(d) I was reading the newspaper in the living room.
(e) The mobile phone is not on the kitchen table.

14) Which set(s) of the following statements is/are consistent?

(a) $p \wedge q,\ p \vee q,\ \sim p$	(b) $p \vee q,\ \sim p,\ \sim q$	(c) $q \Rightarrow p,\ p \Rightarrow \sim r,\ q,\ r$
(d) $q \Leftrightarrow p,\ \sim p,\ \sim q$	(e) $q \Rightarrow p,\ p \Rightarrow \sim r,\ r$	

15) Which of the following is/are logically equivalent to $p \Leftrightarrow q$

(a) $\sim p \vee q$.	(b) $(\sim p \vee q) \wedge (p \vee \sim q)$.
(c) $(p \Rightarrow q) \wedge (q \Rightarrow p)$.	(d) $(\sim p \Rightarrow \sim q) \wedge (\sim q \Rightarrow \sim p)$.
(e) $\sim(\sim p \wedge q)$.	

16) Which of the following arguments is/are valid?

(a) $p \Rightarrow q, p \vdash \sim q$	(b) $p \Rightarrow q, \sim q \vdash \sim p$	(c) $p \Rightarrow q, \sim q \vdash p$
(d) $\sim p \vee q, p \vdash \sim q$	(e) $p \Rightarrow q, p \vdash q$	

17) Let $D=\{x_1, x_2, x_3, ..., x_n\}$. If p(x) is a predicate defined on the set D and $\forall x\ p(x)$ is false, which of the following is/are true?

(a) $p(x_1) \vee p(x_2) \vee .....\vee p(x_n)$ must be true.	(b) $\exists x \sim p(x)$ must be true.
(c) $p(x_1) \vee p(x_2) \vee .....\vee p(x_n)$ must be false.	(d) $\exists x\ p(x)$ must be true.
(e) $p(x_1) \wedge p(x_2) \wedge ..... \wedge p(x_n)$ must be false.	

18) Let $D=\{x_1, x_2, x_3, ..., x_n\}$. If p(x) is a predicate defined on the set D and $\exists x\ p(x)$ is false, which of the following is/are true?

(a) $p(x_1) \vee p(x_2) \vee .....\vee p(x_n)$ must be true.	(b) $\exists x \sim p(x)$ must be true.
(c) $p(x_1) \vee p(x_2) \vee .....\vee p(x_n)$ must be false.	(d) $\exists x\ p(x)$ must be true.
(e) $p(x_1) \wedge p(x_2) \wedge ..... \wedge p(x_n)$ must be false.	

19) Let the two predicates p(x) and q(x) be defined as p(x): $x < 0$ and q(x): $x > 0$ where $x \in R$.

Which of the following propositions is/are true?

(a) $\exists x \sim(p(x) \vee q(x))$.	(b) $(\forall x\ p(x)) \vee (\forall x\ q(x))$.	(c) $\forall x\ (p(x) \vee q(x))$.
(d) $\exists x\ (\sim p(x) \wedge \sim q(x))$.	(e) $(\exists x\ p(x)) \vee (\exists x\ q(x))$.	

20) Consider the following:

(i) Pidurutalagala is the shortest mountain in Sri Lanka.
(ii) What is the difference between propositions and predicates?
(iii) Given any integer y, there is an integer x such that $y < x$.
(iv) $y = x$.

Which of the following is/are correct?

(a) (i) is a proposition.	(b) (ii) is a proposition.	(c) (iii) is a proposition.
(d) (iv) is a proposition.	(e) only (i) is a proposition.	

21) Let p and q be propositions. Which of the following is/are correct?

(a) $(p \wedge \sim p) \vee \sim q$ is a contradiction.	(b) $(p \vee \sim p) \wedge \sim q$ is a contradiction.
(c) $(p \vee \sim p) \wedge q$ is a tautology.	(d) $(p \wedge \sim p) \vee (q \vee \sim q)$ is a tautology.
(e) $(p \vee \sim p) \vee q$ is a tautology.	

22) Let A = {3, 4, 6} , B={1, 2, 8, 9}, $\beta=\{(x,y)|\ x\in A, y\in B, x < y\}$.

Which of the following belong to β?

(a) (3,4).	(b) (4,2).	(c) (6,8).
(d) (1,2).	(e) (4,9).	

23) Let A and B be two non-empty sets and α be a relation defined from A to B.

Which of the following must be true?

(a) $D(\alpha)=A$.	(b) $D(\alpha)\subseteq A$.
(c) $R(\alpha)=B$.	(d) $R(\alpha)\subseteq B$.
(e) $\alpha\subseteq A \times B$.	

24) Let A be the set of all living people and β be the relation defined in A by $\beta=\{(p,q) \mid p \text{ is a parent of } q\}$.

What is β^{-1}?

(a) $\beta^{-1}=\{(p,q)\mid p \text{ is a father of } q\}$.	(b) $\beta^{-1}=\{(p,q)\mid p \text{ is a mother of } q\}$.
(c) $\beta^{-1}=\{(p,q)\mid p \text{ is a brother of } q\}$.	(d) $\beta^{-1}=\{(p,q)\mid q \text{ is a child of } p\}$.
(e) $\beta^{-1}=\{(p,q)\mid p \text{ is a child of } q\}$.	

25) Let α be the relation defined by $\alpha=\{(p,q)|\ p\in Z, q\in Z, (p-q) \text{ is divisible by } 3\}$.

Which of the following must be true?

(a) $\alpha^{-1}=\{(p,q)|\ p\in Z, q\in Z, (q-p) \text{ is divisible by } 3\}$.
(b) α is reflexive.
(c) α is symmetric.
(d) $D(\alpha)=Z$ and $R(\alpha)=Z$.
(e) $D(\alpha)\subset Z$ and $R(\alpha)\subset Z$.

26) Let α and β be two relations defined on A={1,2,3} by

$\alpha=\{(1,1), (1,2), (2,3), (3,1), (3,3)\}$
$\beta=\{(1,2), (1,3), (2,1), (3,3)\}$

Which of the following is/are true?

(a) $\alpha\mathbf{o}\beta = \{(1,2),(1,1),(2,3),(3,2),(3,3)\}$ and $\beta\mathbf{o}\beta=\{(1,1),(1,3),(2,2),(3,3)\}$.
(b) $\alpha\mathbf{o}\beta = \{(1,2),(1,1),(2,3),(3,2),(3,3)\}$ and $\beta\mathbf{o}\beta=\{(1,1),(1,3),(2,2),(2,3),(3,3)\}$.
(c) $\alpha\mathbf{o}\beta = \{(1,3),(1,1),(1,3),(2,1),(2,2),(3,1),(3,3)\}$ and $\beta\mathbf{o}\beta=\{(1,1),(1,3),(2,2),(2,3),(3,3)\}$.
(d) $\alpha\mathbf{o}\beta = \alpha\cup\beta$.
(e) $\alpha\mathbf{o}\beta=\alpha\cap\beta$.

27) Consider the following three relations.

$\beta=\{(x,y)|\ x\in Z,\ y\in Z,\ x = y\}$
$\alpha=\{(x,y)|\ x\in Z,\ y\in Z,\ x \leq y\}$
$\rho=\{(x,y)|\ x\in Z,\ y\in Z,\ x > y\}$

Which of the following is/are true?

(a) β - symmetric, α - symmetric, ρ - transitive.
(b) β - reflexive, α - symmetric, ρ - transitive.
(c) β - transitive, α - transitive, ρ - transitive.
(d) β - symmetric, α - symmetric, ρ - symmetric.
(e) $\alpha \cap \rho = \varnothing$.

28) Let ρ be defined on $A=\{1,2,3\}$ by

$\rho=\{(1,1),(2,2),(3,3),(1,2),(2,1),(2,3),(3,2),(1,3),(3,1)\}$

Which of the following is/are true?

(a) $[1]_\rho=\{2,3\}$, $[2]_\rho=\{1,3\}$, $[3]_\rho=\{1,2\}$.
(b) $[1]_\rho=A$, $[2]_\rho=\{1,3\}$, $[3]_\rho=\{1,2\}$.
(c) $[1]_\rho=A$, $[2]_\rho=\{1,3\}$, $[3]_\rho=A$.
(d) $[1]_\rho=[2]_\rho=[3]_\rho=A$.
(e) $[1]_\rho=\{1\}$, $[2]_\rho=\{2\}$, $[3]_\rho=\{3\}$.

29) Which of the following relations is a/are function(s)?

(a) $\alpha_1=\{(x,y)|\ x\in Z,\ y\in Z,\ x = y\}$.
(b) $\alpha_4=\{(x,y)|\ x\in Z,\ y\in Z,\ y=2x+1\}$.
(c) $\alpha_2=\{(x,y)|\ x\in Z,\ y\in Z,\ x^2 = y^2\}$.
(d) $\alpha_5=\{(x,y)|\ x\in Z,\ y\in Z,\ x < y\}$.
(e) $\alpha_3=\{(x,y)|\ x\in Z,\ y\in Z,\ x \geq y\}$.

30) Let the function $f: Z \longrightarrow Z$ be defined by $f(x) = 2x - 3$.

Which of the following is/are true?

(a) $D(f^{-1}) = Z$, $f^{-1}(x) = 3x - 2$
(b) $D(f^{-1}) = Z$, $f^{-1}(x) = \frac{1}{2}x + 3$
(c) $D(f^{-1}) = Z$, $f^{-1}(x) = \frac{1}{2}(x + 3)$
(d) $D(f^{-1}) = Z$, $f^{-1}(x) = \frac{1}{3}(x + 2)$
(e) $D(f^{-1}) = Z$, $f^{-1}(x) = \frac{1}{2}(x - 3)$

31) Let the functions f and g be defined by $f(x) = 2x - 1$ and $g(x) = x^2 - 1$, where $x\in R$. Then $gof(x)$ equals

(a) $4x^2 - 4$.
(b) $2(x^2 - 1) - 1$.
(c) $2x^2 - 3$.
(d) $4x(x - 1)$.
(e) $(x^2 - 1)(2x - 1)$.

32) Suppose A is a non-empty set and B = { $f \mid f$ is a bijection from A onto A }. Let $f, g \in$ B. Which of the following is/are true?

(a) $f \circ g \in$ B.	(b) $f \circ g = g \circ f$	(c) R(f) $\subset$ B.
(d) $f \circ f^{-1} \in$ B.	(e) $f \circ f^{-1} = f^{-1} \circ f$	

33) In how many ways can the letters of the word SUNDAY be arranged if the arrangement begins with S and ends with Y?

(a) 70	(b) 24	(c) 4**!**
(d) 6**!**	(e) (4**!**)/(2**!**)	

34) Suppose you need to answer 10 out of 13 questions at an examinatioin. How many choices do you have if you must answer the first two questions?

(a) $_{11}P_3$	(b) $_{11}C_3$	(c) $_{11}P_8$
(d) $_{11}C_8$	(e) 165	

35) Which of the following is/are true?

(a) $(x+a)^n = \sum_{r=0}^{n} \binom{n}{r} x^r a^{n-r}$.

(b) $(x+a)^n = a^n + \sum_{r=1}^{n} \binom{n}{r} x^r a^{n-r}$.

(c) $(x+a)^n = \sum_{r=1}^{n} \binom{n}{r} x^r a^{n-r}$

(d) $(x+a)^n = \sum_{r=0}^{n} \binom{n}{n-r} x^r a^{n-r}$.

(e) $(x+a)^n = a^n + x^n + \sum_{r=1}^{n-1} \binom{n}{r} x^r a^{n-r}$.

36) Let the 6-tuple <C, +, *, ′, 0, U > be a Boolean algebra where C is a non-empty set, + and * the sum and the product operators respectively, 0 and U the zero and the unit elements respectively and ′ the complement operator.

If *A*= power set of a non-empty set X,
B^c= the complement of the set B,
P= the set of propositions,

then which of the following is/are a Boolean algebra/s?

(a) <*A*, $\cup$, $\cap$, ^{c} , $\varnothing$, X >.	(b) <*A*, $\cap$, $\cup$, ^{c} , $\varnothing$, X >.
(c) <*P*, $\vee$,$\wedge$, ~ , F, T >.	(d) <*P*, $\wedge$,$\vee$, ~ , F, T >.
(e) <*P*, $\vee$,$\wedge$, $\rightarrow$, F, T >.	

37) Let <C, +, *, ′, 0, U > be a Boolean Algebra as described in question (36).

Which of the following is/are true?

(a) $U * U = U$	(b) $U + U = 2U$
(c) $0'$=U	(d) $U + 0 = U$
(e) $U + 0 = 0$	

38) At an examination, students can select three subjects out of many subjects. The following information is given on the subjects STATS and IT and MATHS.

45% of all students take MATHS
20% take IT and STATS but not MATHS
5% take IT and MATHS but not STAT
90% take at least one of STATS, IT and MATHS
10% take STATS and MATHS but not IT
20% take only MATHS
50% take STAT

The percentage of students who take STATS and MATHS and IT is

(a) 5%	(b) 10%
(c) 15%	(d) 20%
(e) 25%	

39) If the following probabilities were arranged in ascending order, which one would come third?

(a) The probability that a fair die will produce an even number.

(b) A random digit from 1 to 9 (inclusive), is chosen with all digits being equi-probable. The probability that its square will end in one.

(c) The probability that a letter chosen from the alphabet (all letters being equi-probable) will be a vowel.

(d) A random number between 1 and 20 (inclusive) is chosen. The probability that its square root will not be an integer.

(e) A map covers a squared area from co-ordinates (-10, -10) to (+10, +10). A point is chosen at random from the map. The probability that the point chosen will be within 7 units of the origin. [Hint: Area of a circle = $\frac{22}{7} \times \text{radius}^2$]

40) Suppose two gamblers A and B play 10 games of which 5 are won by A, 4 by B and 1 ends in a tie. They agree to play against each other in a tournament consisting of three games. What is the probability that B wins at least one game?

(a) 0.064	(b) 0.216
(c) 0.784	(d) 0.144
(e) 0.875	

41) The chance that you will be caught for illegal parking on campus is 1/3. During the last nine days, you have parked illegally every day and have NOT been caught (you lucky person!). Today, on the 10^{th} day, you again decide to park illegally. The chance that you will be caught is:

(a) greater than 1/3 because you were not caught during the last nine days.
(b) less than 1/3 because you were not caught during the last nine days.
(c) equal to 1/3 because the last nine days do not affect the probability.
(d) equal to 1/10 because you were not caught during the last nine days.
(e) equal to 9/10 because you were not caught during the last nine days.

42) In a shipment of 100 television sets, 6 are defective. If a person buys two televisions from that shipment, what is the probability that both are defective?

(a) 3/100 (b) 9/2500
(c) 12/100 (d) 1/330
(e) 24/100

43) In the United States, 43% of people wear a seat belt while driving. If two people are chosen at random, what is the probability (rounded value) that each of them wears a seat belt?

(a) 86% (b) 57%
(c) 18% (d) 21%
(e) 32%

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