UNIVERSITY OF COLOMBO SCHOOL OF COMPUTING

UNIVERSITY OF COLOMBO, SRI LANKA

DEGREE OF BACHELOR OF INFORMATION TECHNOLOGY (EXTERNAL)

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IT2103: Mathematics for Computing I

09th August, 2008

(TWO HOURS)

Important Instructions :

- The duration of the paper is 2 (two) hours.
- The medium of instruction and questions is English.
- The paper has **45** questions and **9** pages.
- All questions are of the MCQ (Multiple Choice Questions) type.
- All questions should be answered.
- Each question will have 5 (five) choices with **one or more** correct answers.
- All questions will carry equal marks.
- There will be a penalty for incorrect responses to discourage guessing.
- The mark given for a question will vary from 0 (*All the incorrect choices are marked & no correct choices marked*) to +1 (*All the correct choices are marked & no incorrect choices are marked*).
- Answers should be marked on the special answer sheet provided.
- Note that questions appear on both sides of the paper.
- If a page is not printed, please inform the supervisor immediately.
- Mark the correct choices on the question paper first and then transfer them to the given answer sheet which will be machine marked. **Please completely read and follow the instructions given on the other side of the answer sheet before you shade your correct choices.**

Notations:

Z – set of integers	N – set of positive integers
R – set of real numbers	ϕ - (null) empty set
S – Universal set	R^+- set of non-negative real numbers

1) $5\log_3 6-(2\log_3 4+\log_3 54)$ equals

(a) 4.	(b) 2.	(c) 3.
(d) 9.	(e) 6.	

2) $\left(\sqrt[4]{x\sqrt{3}}\right)^{\frac{2}{\sqrt{3}}}\times\left(\sqrt[3]{x\sqrt{3}}\right)^{\frac{3}{\sqrt{12}}}$, $x\in R^+$

equals

(a) $\sqrt{x}$	(b) $x^{\frac{8}{3}}\times x^{\frac{9}{6}}$	(c) x
(d) $x^{\frac{16}{3}}\times x^{\frac{27}{6}}$	(e) $x^{\frac{5}{2}}$	

3) E = {x | x sat the Advanced Level examination in year 2000}
A = {x | x ∈ E and x sat the Biology paper}
B = {x | x ∈ E and x sat the Physics paper}
C = {x | x ∈ E and x sat the Combined Mathematics paper}
D = {x | x ∈ E and x sat the Chemistry paper}
F ={x | x ∈ E and x sat the Higher Mathematics paper}.

If G = {x | x ∈ E and (x sat the combined Mathematics paper) and (x sat the higher Mathematics paper) or (x sat the Chemistry paper)} and
H = {x | x ∈ E and x sat the Biology paper and the Physics paper}, then,

(G, H) equals

(a) (C ∩ F ∩ D, A ∪ B).	(b) (C ∩ F ∩ D, A ∩ B).
(c) (C ∩ (F ∪D), A ∪ B).	(d) (C ∩ (F ∪ D), A ∩ B).
(e) ((C ∩ F) ∪ (C ∩ D), A ∩ B).	

4) A = {x | x ∈ R and $2x^2 + 3x = 0$}. Then A equals

(a) {0}.	(b) {3/2}.	(c) {0, -3/2}.
(d) {0, 3/2}.	(e) {-3/2}.	

5) The sets A, B, C are such that A is a subset of $B \cup C$. Now consider the following.
(α) A is a subset of B. (β) A is a subset of C. (γ) $A \cap (B \cup C) = A$.

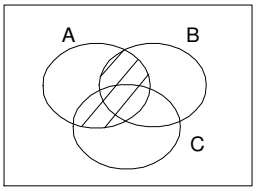
Which of the following are(is) true?

(a) All of (α), (β), (γ) are true.	(b) None of (α), (β), (γ) is true.
(c) Only (α) is true.	(d) Only (β) is true.
(e) Only (γ) is true.	

6) Which of the following are(is) **not** true?

(a) $N \cap Z = N$.	(b) $N \cup Z = Z$.	(c) N is a subset of Z.
(d) $N \cup Z = Z \cup N$.	(e) $Z \setminus N = N \setminus Z$.	

7)



The shaded portion in the Venn diagram represents

(a) $A \cap (B \cup C)$.	(b) $B \cap (A \cup C)$.	(c) $C \cap (A \cup B)$.
(d) $A \cap C$.	(e) $B \cap A$.	

8) $A = \{x \mid x \in Z, x \text{ is a multiple of 2 and } 1 < x \leq 14\}$
$B = \{x \mid x \in Z, x \text{ is a multiple of 3 and } 1 < x \leq 15\}$.

$A \cap B$ equals

(a) $\{x \mid x \in Z$ and x is odd and $1 < x \leq 14\}$.	(b) $\{6, 12\}$.
(c) $\{6, 12, 15\}$.	(d) $\{3, 6, 9, 12\}$.
(e) $\{x \mid x \in Z$ and x is a multiple of 6 and $1 < x \leq 14\}$.	

9) Consider the following:
(i) This sentence has five words.
(ii) Where are you going?
(iii) $1 > 2$.
(iv) There is a real number y such that for any given real number x, $y > x$.
(v) $y > x$.

Which of the following are(is) correct?

(a) (i) is a proposition.	(b) (ii) is a proposition.	(c) (iii) is a proposition.
(d) (iv) is a proposition.	(e) (v) is a proposition.	

10) Out of the 16 possible sets of truth values for p, q, r, s,
$(p \vee q) \Leftrightarrow (r \wedge s)$ is <u>true only for</u>

(a) 12 sets of values.	(b) 10 sets of values.	(c) 9 sets of values.
(d) 8 sets of values.	(e) 6 sets of values.	

11) Consider the following truth table.

p	q	r	α_1	α_2	α_3
T	T	T	T	T	T
T	T	F	F	F	F
T	F	T	F	F	F
T	F	F	F	F	F
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	T
F	F	F	T	F	F

Now, $(p \Rightarrow q) \wedge r$ is one of $\alpha_1, \alpha_2, \alpha_3$ and $p \Rightarrow (q \wedge r)$ is also one of $\alpha_1, \alpha_2, \alpha_3$.
Which of the following are(is) correct?

(a) $(p \Rightarrow q) \wedge r$ is α_1 and $p \Rightarrow (q \wedge r)$ is α_2.	(b) $(p \Rightarrow q) \wedge r$ is α_2 and $p \Rightarrow (q \wedge r)$ is α_1.
(c) $(p \Rightarrow q) \wedge r$ is α_2 and $p \Rightarrow (q \wedge r)$ is α_3.	(d) $(p \Rightarrow q) \wedge r$ is α_1 and $p \Rightarrow (q \wedge r)$ is α_3.
(e) $(p \Rightarrow q) \wedge r$ is α_3 and $p \Rightarrow (q \wedge r)$ is α_2.	

12) Consider the following:
(i) If I am awake, then I work hard.
(ii) I dream only if I am not awake.
(iii) Working hard is sufficient for me to be awake.
(iv) Being awake is necessary for me not to dream.
Now:
'I am awake' is denoted by one of p, not p, q, not q, r, not r.
'I work hard' is denoted by one of p, not p, q, not q, r, not r.
'I dream' is denoted by one of p, not p, q, not q, r, not r.

Which of the following are(is) correct?

(a) (i) means $r \Rightarrow$ (not p)	(ii) means (not r) $\Rightarrow p$	(iii) means $p \Rightarrow q$	(iv) means $q \Rightarrow p$.
(b) (i) means $p \Rightarrow q$	(ii) means (not r) $\Rightarrow q$	(iii) means $q \Rightarrow p$	(iv) means $r \Rightarrow$ (not p)
(c) (i) means $q \Rightarrow p$	(ii) means $r \Rightarrow$ (not q)	(iii) means $q \Rightarrow p$	(iv) means $r \Rightarrow$ (not p)
(d) (i) means (not r) $\Rightarrow p$	(ii) means $r \Rightarrow$ (not p)	(iii) means $p \Rightarrow q$	(iv) means $q \Rightarrow p$
(e) (i) means $p \Rightarrow q$	(ii) means $r \Rightarrow$ (not p)	(iii) means $q \Rightarrow p$	(iv) means (not r) $\Rightarrow p$

13) It is given that $(p \wedge q) \Rightarrow r$ is true. Then which of the following are(is) necessarily true?

(a) $p \Rightarrow r$	(b) $q \Rightarrow r$	(c) $((\text{not } p) \wedge (\text{not } q)) \Rightarrow r$
(d) $(\text{not } r) \Rightarrow ((\text{not } p) \wedge (\text{not } q))$	(e) $(\text{not } r) \Rightarrow ((\text{not } p) \vee (\text{not } q))$	

14) Consider the following:
(i) $x > y+z$. (ii) $x > 0$. (iii) $1 = 2$. (iv) Given any real number x, $x < y$.

Which of the following are(is) correct?

(a) (i) is a proposition.	(b) (ii) is a proposition.
(c) (iii) is a proposition.	(d) (iv) is a proposition.
(e) (i), (ii), (iii) and (iv) are not propositions.	

15) Consider the following:
(i) For every positive integer n, $2^n \geq n^2$
(ii) For every real number x, $x^4 = 1 \Rightarrow x = 1$ or $x = -1$
(iii) For every positive integer n, $n^2 = 4 \Rightarrow n = 2$.

Which of the following are(is) correct?

(a) (i) is true.	(b) (ii) is true.	(c) (iii) is true.
(d) (i) is false.	(e) (iii) is false.	

16) Let A = {1, 2, 3} be the universal set. Now consider the following:
(i) $\exists x \, \forall y, x^2 < y$ (ii) $\forall x \, \exists y, x^2 + y^2 < 12$ (iii) $\forall x \, \forall y, x^2 + y^2 < 12$

Which of the following are(is) correct?

(a) (i) is true	(b) (ii) is true	(c) (iii) is true
(d) (i) is false	(e) (iii) is false	

17) For $n \in N$, P(n) is a proposition such that for any $n \in N$, $P(n) \Rightarrow P(n + 1)$ and, for any $n \in N$, $P(n + 1) \Rightarrow P(n)$ are true.

Which of the following are(is) true?

(a) For any $n \in N$, $P(n) \Leftrightarrow P(n+1)$.	(b) For any $n \in N$, P(n).
(c) For any $n \in N$, P(n + 1).	(d) For any $n \in N$, $P(5) \Rightarrow P(n)$.
(e) For any $n \in N$, $P(1) \Rightarrow P(n)$.	

18) Let p, q and r be propositions. Which of the following are(is) correct?

(a) $(p \wedge q) \wedge r$ is equivalent to $p \wedge (q \wedge r)$.	(b) $(p \vee q) \wedge r$ is equivalent to $p \vee (q \wedge r)$.
(c) $p \wedge (p \vee q)$ is equivalent to q.	(d) $p \wedge (p \vee q)$ is equivalent to p.
(e) $p \wedge (q \vee r)$ is equivalent to $(p \vee q) \wedge (p \vee r)$.	

19) Let p, q and r be propositions. Which of the following are(is) correct?

(a) p is a Tautology.	(b) $p \wedge$ not p is a Contradiction.
(c) $p \wedge$ not p is a Tautology.	(d) $p \wedge q \Rightarrow p$ is a Tautology.
(e) $(p \Rightarrow q) \wedge q \Rightarrow (p \Rightarrow q)$ is a Tautology.	

20) Let S=Z be the universal set. Now consider the following:
(i) $\exists x, x > 1 \wedge x < 1$. (ii) $(\exists x, x > 1) \wedge (\exists x, x < 1)$ (iii) $(\exists x, x > 1) \vee (\exists x, x < 1)$

Which of the following are(is) correct?

(a) (i) is true.	(b) (ii) is true.	(c) (iii) is true.
(d) (ii) is false.	(e) (iii) false.	

21) Let A={2,4,6}, B{3,4,5,7,8} and the relation ρ be defined from A to B as follows:

ρ={ (x,y)| x divides y}. What are D(ρ) and R(ρ) in the usual notation?

(a) D(ρ)= { 2,4,6}, R(ρ)={4,8}.	(b) D(ρ)= { 4,8}, R(ρ)={2,4,6}.
(c) D(ρ)= { 2,4}, R(ρ)={4,8}.	(d) D(ρ)= { 4,8}, R(ρ)={2,4}.
(e) D(ρ)= { 3,4}, R(ρ)={4,6}.	

22) Let A={2,4}, B={3,5} and C={3,9,11} be three sets. Which of the following are(is) correct?

(a) A x B = {(2,3),(4,5)}.	(b) A x (B ∩ C) = {(2,3),(4,3)}.
(c) B x A = {(3,2),(3,4),(5,2),(5,4)}.	(d) (B ∩ C) x (A ∪ B) ={(3,2),(3,3),(3,4),(3,5)}.
(e) C x (A ∩ B) = (A ∩ C) x B.	

23) Let A={2,4,6} and the relation $\boldsymbol{\rho}$={(2,2), (2,4), (4,4), (4,2),(6,4),(4,6),(6,6)}. Which of the following are(is) correct?

(a) ρ is reflexive.	(b) ρ is symmetric.	(c) ρ is transitive.
(d) ρ is **not** symmetric.	(e) ρ is **not** transitive.	

24) Suppose an equivalence relation ρ defined on A={1,2,3,4,5} is given as

$\boldsymbol{\rho}$={ (1,1), (2,2), (3,3), (4,4), (5,5), (1,2), (2,1), (2,3), (3,2), (1,3), (3,1), (4,5), (5,4) }

What is $[1]_\rho \cup [4]_\rho$?

(a) {2,3,4,5}.	(b) {2,3,5}.	(c) {1,4}.
(d) A.	(e) {1,2,3}.	

25) Let A={1,2,3} and *f* be a function defined on A. Which of the following would correctly represent *f* ?

(a) $f(1)=10, f(2)=10, f(3)=10$.	(b) $f(1)=8, f(2)=8, f(3)=10$.
(c) $f(1)=7, f(1)=8, f(2)=9, f(3)=10$.	(d) $f(1)=8, f(2)=9$.
(e) $f(1)=1, f(2)=2, f(3)=3$.	

26) Suppose f is a 1-1 function and $x,y \in D(f)$. Which of the following are(is) correct about f ?

(a) $\forall x \forall y\ x \in D(f), y \in D(f),\ x = y \Rightarrow f(x) = f(y)$.
(b) $\forall x \forall y\ x \in D(f), y \in D(f),\ x \neq y \Rightarrow f(x) \neq f(y)$.
(c) $\forall x \forall y\ x \in D(f), y \in D(f),\ f(x) = f(y) \Rightarrow x = y$.
(d) $\forall x \forall y\ x \in D(f), y \in D(f),\ f(x) \neq f(y) \Rightarrow x \neq y$.
(e) $\forall x \forall y\ x \in D(f), y \in D(f)\ f(x) = f(y)$.

27) Suppose h is a 1-1 function. Which of the following are(is) correct about the inverse function h^{-1} of h?

(a) $D(h^{-1}) = R(h)$. (b) $R(h^{-1}) = D(h)$. (c) h^{-1} is not 1-1.
(d) $D((h^{-1})^{-1}) = R(h)$. (e) h^{-1} is 1-1.

28) Suppose three functions, f, g and h are defined as follows

$D(f)=\{1,2,3\}$, $f(1)=10$, $f(2)=11$, $f(3)=12$
$D(g)=\{4,5,6\}$, $g(4)=3$, $g(5)=1$, $g(6)=0$
$D(h)=\{7,8,9\}$, $h(7)=5$, $h(8)=6$, $h(9)=4$

Then $f \circ g \circ h$ is

(a) $D(f \circ g \circ h)=\{1,3\}$, $f \circ g \circ h(1)=10$, $f \circ g \circ h(3)=12$
(b) $D(f \circ g \circ h)=\{4,5\}$, $f \circ g \circ h(4)=12$, $f \circ g \circ h(5)=10$
(c) $D(f \circ g \circ h)=\{7,9\}$, $f \circ g \circ h(7)=10$, $f \circ g \circ h(9)=12$
(d) $D(f \circ g \circ h)=\{7,8,9\}$, $f \circ g \circ h(7)=10$, $f \circ g \circ h(8)=0$, $f \circ g \circ h(9)=12$
(e) $D(f \circ g \circ h)=\{1,2,3\}$, $f \circ g \circ h(1)=5$, $f \circ g \circ h(2)=6$, $f \circ g \circ h(3)=4$

29) Suppose A is a set and $B = \{ f \mid f \text{ is a bijection from A to A} \}$. If $f,g \in B$, which of the following are(is) correct?

(a) f is 1-1 and into. (b) f is onto. (c) $f \circ g \in B$.
(d) $f^{-1} \in B$. (e) $f^{-1} \circ f \notin B$.

30) How many distinct permutations can be formed from all the letters of the word "CHAIR"?

(a) 1. (b) 60. (c) 30.
(d) 90. (e) 120.

31) How many distinct permutations can be formed from all the letters of the word "TOFFEE"?

(a) 1. (b) 15. (c) 30.
(d) 45. (e) 60.

32) Suppose 2 fair dice are thrown once. Let A be the event that the sum of the values appearing on top of the 2 dice is 8. Then P(A) =

(a) 1/12.	(b) 1/18.	(c) 1/9.
(d) 1/6.	(e) 5/36.	

33) In how many ways can 3 boys and 4 girls sit in a row if all the boys are to sit together?

(a) 3! . 4!	(b) 2 . 3! . 4!	(c) 5! . 3!
(d) 2. 5! . 3!	(e) 4! . 5!	

34) Lets assume that you play a game that involves tossing of three fair coins at the same time. If all the coins come up the same sides (all heads or tails) you win the game. What is the probability that you will lose the game?

(a) 1/4.	(b) 2/4.	(c) 3/4.
(d) 1/8.	(e) 5/8.	

35) To arrange in 24 different ways around a circular table, how many boys should be there, if all of them participate in each arrangement?

(a) 3.	(b) 4.	(c) 5.
(d) 6.	(e) 7.	

36) In how many ways can 4 girls and 2 boys sit at a movie theatre row with 6 seats if a girl must be seated at each end?

(a) 144.	(b) 288.	(c) 24.
(d) 720.	(e) 24.	

37) In how many ways can a committee of 5 be chosen from 10 boys and 8 girls so that the number of girls would be more than the number of boys in the committee?

(a) 252.	(b) 56.	(c) 14112.
(d) 2520.	(e) 3271.	

38) Which of the following number(s) **cannot** be a probability of some event?

(a) –0.2	(b) 0.5	(c) 0
(d) 1	(e) 1.25	

39) A jar contains 3 red marbles, 7 green marbles and 10 white marbles. If a marble is drawn from the jar at random, what is the probability that this marble is white?

(a) 1/10.	(b) 1/20.	(c) 10/20.
(d) 3/20.	(e) 0.	

40) A bag contains 5 red marbles, 6 blue marbles and 7 yellow marbles. What is the probability that the first marble taken out of the bag is red or blue?

(a) $1/(6 \times 7)$.	(b) $(6 \times 7)/18$.	(c) $(5/18) \times (6/18)$.
(d) $(5/18) + (6/18)$.	(e) 0.	

41) A box has three drawers; one contains two gold coins, one contains two silver coins, and one contains one gold coin and one silver coin. Assume that one drawer is selected randomly and that a randomly selected coin from that drawer is gold. What is the probability that the chosen drawer is the one that contains two gold coins?

(a) 1/3.	(b) 2/3.	(c) 1/6.
(d) 1/2.	(e) ¼.	

42) A book contains 732 pages numbered 1, 2, ..., 732. If a student randomly opens the book, what is the probability that the page number contains the digit 1?

(a) 112/732.	(b) 184/732.	(c) 227/732.
(d) 233/732.	(e) 254/732.	

43) A card is drawn at random from a deck with 52 cards and then, without replacing the first card, a second card is drawn from the same deck. What is the probability that both cards will be aces?

(a) (4/52) + (3/52).	(b) $(4/52) \times (3/52)$.	(c) (4/52) + (3/51).
(d) $(4/52) \times (3/51)$.	(e) $(1/52) \times (1/51)$.	

44) Let $(\{0,1\}, \wedge, \vee, ' , 0, 1)$ be a Boolean algebra with the following properties

- $0 \wedge 0 = 0$
- $1 \wedge 1 = 1$
- $0 \vee 0 = 0$
- $0' = 1$

Which of the following is/are correct?

(a) $0 \neq 1$.	(b) $1 \wedge 0 = 1$.	(c) $1 \vee 1 = 1$.
(d) $1 \vee 1 = 0$.	(e) $1' = 0$.	

45) Let $(X, *, +, ' , 0, 1)$ be a Boolean algebra. Where $0' = 1$ and $1'=0$.
If A is an element of the set X, what is the dual of the Boolean expression $A + 1 = 1$?

(a) $A * 1 = 1$.	(b) $A * 0 = 0$.	(c) $A + 0 = 0$.
(d) $A * A = A$.	(e) $A * 1 = 1$.	

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