UNIVERSITY OF COLOMBO, SRI LANKA

UNIVERSITY OF COLOMBO SCHOOL OF COMPUTING

(Successor to the Institute of Computer Technology (ICT), University of Colombo)

DEGREE OF BACHELOR OF INFORMATION TECHNOLOGY (EXTERNAL)

Academic Year 2003/2004 – 1st Year Examination – Semester 1

IT1102: Mathematics for Computing I

14th February 2004

(TWO HOURS)

Important Instructions :

- The duration of the paper is 2 (two) hours.
- The medium of instruction and questions is English.
- The paper has **45** questions and **9** pages.
- All questions are of the MCQ (Multiple Choice Questions) type.
- All questions should be answered.
- Each question will have 5 (five) choices with **one or more** correct answers.
- All questions will carry equal marks.
- There will be a penalty for incorrect responses to discourage guessing.
- The mark given for a question will vary from –1 (*All the incorrect choices are marked & no correct choices marked*) to +1 (*All the correct choices are marked & no incorrect choices are marked*).
- Answers should be marked on the special answer sheet provided.
- Note that questions appear on both sides of the paper.
- If a page is not printed, please inform the supervisor immediately.
- Mark the correct choices on the question paper first and then transfer them to the given answer sheet which will be machine marked. **Please completely read and follow the instructions given on the other side of the answer sheet before you shade your correct choices.**

Notations:

Z – set of integers N – set of positive integers
R – set of real numbers ϕ - empty set
S – Universal set

1) In how many ways can 3 Sri Lankans, 3 Indians, and 3 Chinese be seated in a row so that those of the same nationality sit together?

(a) 4(3!)	(b) $\frac{3!}{4}$	(c) $\frac{9!}{3!3!3!}$
(d) $(3!)^4$	(e) 3(4!)	

2) How many distinct permutations can be formed from all the letters of the word "COUNT"?

(a) 12	(b) 24	(c) 120
(d) 20	(e) 18	

3) How many distinct permutations can be formed from the letters of the name "MISSISSIPPI"?

(a) $\frac{11!}{4!}$	(b) $\frac{11!}{4!\times 4!\times 2!}$	(c) $\frac{11!}{2!}$
(d) 11!	(e) $\frac{11!}{4!\times 2!}$	

4) In how many ways can 5 persons arrange themselves around a circular table?

(a) 120	(b) 24	(c) 5
(d) 720	(e) 60	

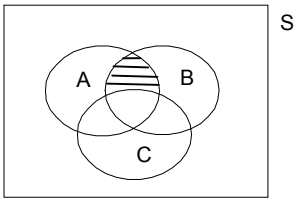
5) The r^{th} term of the expansion of $(a + b)^n$ in ascending powers of a or b can be

(a) $\binom{n}{r}a^r b^{n-r}$.	(b) $\binom{n}{r-1}a^{n-r+1}b^{r-1}$.	(c) $\binom{n}{r}a^{n-r}b^r$.
(d) $\binom{n}{r+1}a^{r+1}b^{n-r-1}$.	(e) $a^{n-r+1}b^{r-1}$.	

6) A woman has 10 close friends. In how many ways can she invite 6 of them to dinner?

(a) 30	(b) 90	(c) 720
(d) 210	(e) 60	

7) Consider the following Venn diagram.



The shaded area in the above diagram corresponds to the event

(a) $A \cap B \cap C$.	(b) $C^c \cap (A \cap B)$.	(c) $B \cap A^c \cap C$.
(d) $(A \cap B)^c \cap C$.	(e) $A \cap B \cap C^c$.	

8) Suppose a fair die is tossed once.
Let A = {odd number occurs} and B = {prime number occurs}.
Then $P(A \cup B)$ =

(a) $\frac{1}{3}$.	(b) $\frac{2}{3}$.	(c) $\frac{1}{6}$.
(d) $\frac{5}{6}$.	(e) $\frac{3}{6}$.	

9) Suppose 2 fair dice are tossed once. Let A be the event that the same number appears on top of both dice, and B be the event that numbers appearing on both dice are odd. Then $P(A \cap B)$ =

(a) $=\frac{1}{6}$.	(b) $=\frac{1}{3}$.	(c) $=1$.
(d) $=\frac{1}{12}$.	(e) $=\frac{1}{2}$.	

10) A box contains 4 red balls, 3 white balls, and 5 black balls. If a ball is taken from the box at random, the probability that the ball is red or black =

(a) $=\frac{1}{4}$.	(b) $=\frac{1}{2}$.	(c) $=\frac{3}{4}$.
(d) $=\frac{5}{36}$.	(e) $=\frac{12}{36}$.	

11) A pair of fair dice is tossed once. If the sum of the two numbers is greater than 9, the probability that one of the numbers is a 6 =

(a) $\frac{1}{6}$.	(b) $\frac{2}{3}$.	(c) $\frac{1}{2}$.
(d) $\frac{1}{3}$.	(e) $\frac{5}{6}$.	

12) In a certain town, 50% of the people subscribe to newspaper A, 60% of the people subscribe to newspaper B and 20% subscribe to both. A person is selected at random.
The probability that he does not subscribe to A given that he does not subscribe to B =

(a) $\frac{1}{10}$.	(b) $\frac{1}{4}$.	(c) $\frac{9}{10}$.
(d) $\frac{1}{2}$.	(e) $\frac{3}{4}$.	

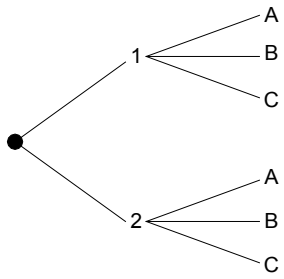
13) A table tennis player has probability $\frac{1}{2}$ of winning (W) and $\frac{1}{2}$ of losing (L). Find the probability that he wins a series of 3 games. (Note that if he wins the first 2, he wins the series and the game stops. Otherwise, he has to win one of the first two and the third to win the series).

(a) $\frac{3}{8}$	(b) $\frac{1}{4}$	(c) $\frac{1}{8}$
(d) $\frac{1}{2}$	(e) $\frac{5}{8}$	

14) Suppose an item code has 6 places of which the first 2 are selected from the 3 letters A, B, C with repetitions allowed, and the last 4 are selected from the 10 digits 0 – 9 with repetitions allowed. The total number of codes will be

(a) $\frac{10!}{5!}$.	(b) 6,000.	(c) $\frac{9 \times 10!}{6!}$.
(d) 90,000.	(e) 10,000.	

15) Consider the following tree diagram :



It shows the possible outcomes of the product set

(a) A x B x C where A = {1, 2}.	(b) A x B x C where each set = {1, 2}.
(c) {1, 2} x {A, B, C}.	(e) {A, B, C} x {A, B, C}.
(e) {1, 2} x {A, B}.	

16) Let the sets A and B be defined as
A = {5,7,9}, B = {x | x is an integer and x > 0}

Which of the following is/are true?

(a) $5 \in B$.	(b) $9 \notin B$.
(c) A = B.	(d) $A \subseteq B$.
(e) $B \subseteq A$.	

17) Let the set A be defined as
A = {x | 3x = 6 }
which of the following is/are true?

(a) A = 2.	(b) A = {2}.
(c) The number of elements in A is 2.	(d) $\phi \subseteq A$.
(e) A = ϕ.	

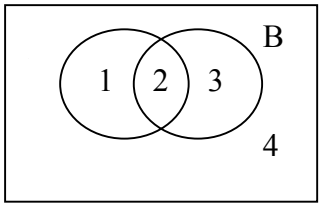
18) Which of the following is/are true?

(a) $\phi = \{0\}$.	(b) $\phi = \{\ \}$.	(c) $\phi = \{\phi\}$.
(d) $\phi \subseteq S$.	(e) There can be only one null set ϕ.	

19) A, B are subsets of a universal set S and A is a subset of B and $A \neq B$. Which of the following statements is/are true?

(a) $B^c \cap A = \phi$.	(b) $A \cap B \neq B$.	(c) $A \cap A^c = \phi$.
(d) $A \cap B = A$.	(e) B is a subset of $A \cap B$.	

20)



The region labelled 4 in the above Venn diagram represents

(a) $A \cap B$.	(b) $A^c \cap B^c$.	(c) $A^c \cup B^c$.
(d) $A \cup B$.	(e) $A^c \cap B$.	

21) If E is any event such that P(E) > 0 and A and B are mutually exclusive, then which of the following is true?

(a) $A \cap E$ and $B \cap E$ are mutually exclusive.	(b) $(A \cup B) \cap E = (A \cap E) \cup (B \cap E)$.
(c) $P(A \cup B \mid E) = P(A \mid E) + P(B \mid E)$.	(d) $P(A \cap B \mid E) = P(E)$.
(e) $P(A \mid E) = P(B \mid E)$.	

22) Which of the following are relations from the set A = {a,b,c} to the set B = {1,2}?

(a) R1 = {(a,1),(b,2),(c,2)}.	(b) R2 = {}.
(c) R3 = A × B.	(d) R4 = {(a,1),(2,b)}.
(e) R5 = {(b,1)}.	

23) Let R be the relation on N, i.e. the set of natural (or counting) numbers $\{1, 2, 3, \ldots\}$, given by $R = \{(x, y) \mid x, y \in N, \text{ and } 3x + y = 15\}$. Which of the following is(are) correct?

(a) $R = \{(1, 12), (2, 9), (3, 6), (4, 3)\}$.
(b) The relation R consists of an infinite number of ordered pairs.
(c) The cardinal number $|R|$ of R is 5.
(d) $R^{-1} = \{(3, 4), (6, 3), (9, 2), (12, 1)\}$.
(e) $R^{-1} = \{(1, 5), (2, 9), (3, 8), (4, 6)\}$.

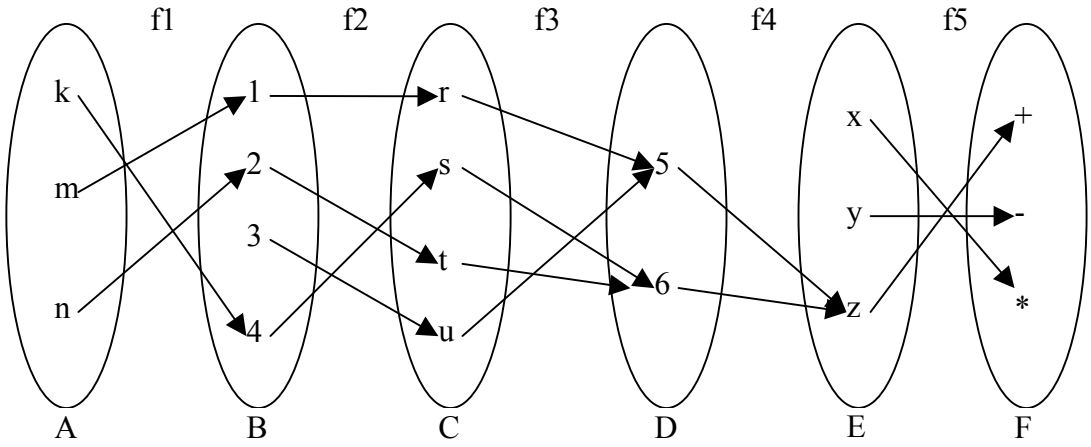
24) Let A = {1,3} and a relation R be defined on A as R = {(1,1), (1,3), (3,1), (3,3)}. Which of the following is/are correct?

(a) R is symmetric.	(b) R is not symmetric.	(c) R is transitive.
(d) R is not transitive.	(e) R is reflexive.	

25) Let A be the set of integers and a relation ~ be defined on A × A as
(a, b) ~ (c, d) if a + d = b + c.
Which of the following is/are correct?

(a) ~ is an equivalence relation.	(b) ~ is not an equivalence relation.
(c) ~ is reflexive.	(d) ~ is symmetric.
(e) ~ is transitive.	

Questions 26, 27 and 28 are based on the functions f1: A $\rightarrow$ B, f2 : B $\rightarrow$ C, f3:C $\rightarrow$ D, f4 : D $\rightarrow$ E and f5: E $\rightarrow$ F defined by the following diagram



26) Which of the following is/are correct?

(a) f1 is an onto function.	(b) f2 is an onto function.	(c) f3 is an onto function.
(d) f4 is an onto function.	(e) f5 is an onto function.	

27) Which of the following is/are correct?

(a) f1 is a one-to-one function.	(b) f2 is a one-to-one function.	(c) f3 is a one-to-one function.
(d) f4 is a one-to-one function.	(e) f5 is a one-to-one function.	

28) Which of the following is/are correct?

(a) f2 and f3 are invertible functions.	(b) f2 and f4 are invertible functions.
(c) f2 and f5 are invertible functions.	(d) f3 and f4 are invertible functions.
(e) f3 and f5 are invertible functions.	

29) Let the function f: R $\rightarrow$ R be defined by the rule f(x) = 5x – 3. What is the inverse function f^{-1} of f?

(a) $f^{-1}(x) = (x - 3)/5$.	(b) $f^{-1}(x) = (x + 3)/5$.	(c) $f^{-1}(x) = (x - 5)/3$.
(d) $f^{-1}(x) = (x + 5)/3$.	(e) $f^{-1}(x) = (x/5) + 3$.	

30) Which of the following is/are correct?

(a) if f: X $\rightarrow$ Y and g: Y $\rightarrow$ Z are both one-to-one functions, then their composite function g $\bullet$ f is one-to-one.

(b) if f: X $\rightarrow$ Y and g: Y $\rightarrow$ Z are both onto functions, then their composite function g $\bullet$ f is onto.

(c) if f: X $\rightarrow$ Y and g: Y $\rightarrow$ Z, then g $\bullet$ f = f $\bullet$ g always.

(d) Inverse function exists for any function.

(e) Inverse of a function exists if it is one-to-one and onto.

31) Consider the following:

(i) If I am awake, then I work hard.
(ii) I dream only if I am not awake.
(iii) Working hard is sufficient for me to be awake.
(iv) Being awake is necessary for me not to dream.

Now:
'I am awake' is denoted by one of p, not p, q, not q, r, not r.
'I work hard' is denoted by one of p, not p, q, not q, r, not r.
'I dream' is denoted by one of p, not p, q, not q, r, not r.

Which one of the following could be correct?

(a) (i) means $r \Rightarrow (\text{not } p)$.	(ii) means $(\text{not } r) \Rightarrow p$.	(iii) means $p \Rightarrow q$.	(iv) means $q \Rightarrow p$.
(b) (i) means $p \Rightarrow q$.	(ii) means $(\text{not } r) \Rightarrow q$.	(iii) means $q \Rightarrow p$.	(iv) means $r \Rightarrow (\text{not } p)$.
(c) (i) means $q \Rightarrow p$	(ii) means $r \Rightarrow (\text{not } q)$.	(iii) means $q \Rightarrow p$.	(iv) means $r \Rightarrow (\text{not } p)$.
(d) (i) means $(\text{not } r) \Rightarrow p$.	(ii) means $r \Rightarrow (\text{not } p)$.	(iii) means $p \Rightarrow q$.	(iv) means $q \Rightarrow p$.
(e) (i) means $p \Rightarrow q$.	(ii) means $r \Rightarrow (\text{not } p)$.	(iii) means $q \Rightarrow p$.	(iv) means $(\text{not } r) \Rightarrow p$.

32) Consider the following:

(i) This sentence has in all ten words.
(ii) Come here!
(iii) Given any real number x, there is a real number y such that $y > x$.
(iv) There is a real number y such that for any given real number x, $y > x$.

Which of the following is/are correct?

(a) (i), (ii), (iii) and (iv) are all propositions.
(b) (i), (ii), (iii) and (iv) are all not propositions.
(c) (iii) is a proposition.
(d) (ii) is a proposition.
(e) (i), (iii) and (iv) are propositions.

33) Consider the following truth tables.

(i)

p	q	$p \vee (p \wedge q)$
T	T	T
T	F	T
F	T	T
F	F	F

(ii)

p	q	$p \wedge (p \vee q)$
T	T	T
T	F	T
F	T	F
F	F	F

(iii)

p	q	$(p \vee q) \Rightarrow (p \wedge q)$
T	T	T
T	F	F
F	T	T
F	F	T

Which of the following is/are correct?

(a) (i), (ii) and (iii) are all correct.
(b) (i), (ii) and (iii) are all incorrect.
(c) (i) is correct but (ii) and (iii) are incorrect.
(d) (ii) is correct but (i) and (iii) are incorrect.
(e) (iii) is correct but (i) and (ii) are incorrect.

34) Out of the 16 possible sets of truth values for p, q, r, s,
$(p \vee q) \Rightarrow (r \vee s)$ is true only for

(a) 8 sets of truth values.
(b) 9 sets of truth values.
(c) 10 sets of truth values.
(d) 12 sets of truth values.
(e) 13 sets of truth values.

35) Consider the following:

(i) $p \wedge (\text{not } (q \wedge r))$ (ii) $(p \wedge (\text{not } q)) \vee (p \wedge (\text{not } r))$ (iii) $(p \wedge q) \Rightarrow r$

(iv) $p \Rightarrow (q \Rightarrow r)$ (v) $(p \wedge q) \vee r$ (vi) $(p \vee q) \wedge r$

Which of the following is/are correct?

(a) (i), (ii) are equivalent.	(b) (i), (ii) are not equivalent.
(c) (iii), (iv) are equivalent.	(d) (iii), (iv) are not equivalent.
(e) (v), (vi) are equivalent.	

36) Let S = {1, 2, 3} be the universal set. Now consider the following:

(i) $\exists x \, \forall y, \, x^2 < y + 1$ (ii) $\forall x \, \exists y, \, x^2 + y^2 < 12$ (iii) $\forall x \, \forall y, \, x^2 + y^2 < 12$

Which of the following is/are correct?

(a) (i), (ii) are true but (iii) is false.
(b) (i) is true but (ii), (iii) are false.
(c) (ii), (iii) are true but (i) is false.
(d) (i), (ii), (iii) are all false.
(e) (i), (ii), (iii) are all true.

37) A = {5, 7, 10}, B = {1, 5, 9}. Now consider the following:

(i) There is $y \in B$ such that for every $x \in A$, $x = y \vee (y < x < y + 4)$.

(ii) For any given $x \in A$, there is a corresponding $y \in B$ such that $x = y \vee (y < x < y + 4)$.

(iii) There is $x \in A$ and $y \in B$ such that $x + y > 12$.

Which of the following is/are correct?

(a) (i) is correct, (ii), (iii) are incorrect	(b) (ii) is correct, (i), (iii) are incorrect
(c) (ii), (iii) are correct, (i) is incorrect	(d) (iii) is correct, (i), (ii) are incorrect
(e) (i), (ii), (iii) are all correct	

38) For $n \in N$, P(n) is a proposition such that for any $n \in N$, $P(2n - 1) \Rightarrow P(2n + 1)$ is true.

Which of the following is/are true?

(a) For any $n \in N$, $P(1) \Rightarrow P(2n - 1)$.
(b) For any $n \in N$, $P(1) \Rightarrow P(n)$.
(c) For any $n \in N$, P(n).
(d) For any $n \in N$, $(P(1) \vee P(2)) \Rightarrow P(2n - 1)$.
(e) For any $n \in N$, $(P(1) \wedge P(2)) \Rightarrow P(2n - 1)$.

39) Denote $(p \wedge (\text{not } q) \wedge r) \vee (p \wedge q \wedge (\text{not } r)) \vee (p \wedge (\text{not } q) \wedge (\text{not } r))$ by α.
Which of the following is/are correct?

(a) α is true when p is true, q is true, r is true.	(b) α is true when p is true, q is true, r is false.
(c) α is true when p is false, q is true, r is true.	(d) α is true when p is true, q is false, r is true.
(e) α is true when p is false, q is false, r is false.	

40) Which of the following is/are correct ?

(a) $(p \vee (\text{not } p)) \Rightarrow (p \wedge (\text{not } p))$ is a contradiction.
(b) $p \vee \text{not } (p \wedge q)$ is a tautology.
(c) $p \vee \text{not } (p \wedge q)$ is a contradiction.
(d) $(p \wedge q) \wedge \text{not } (p \vee q)$ is a tautology.
(e) $(p \wedge q) \wedge \text{not } (p \vee q)$ is a contradiction.

41) For $n \in N$, $P(n)$ is a proposition such that for any $n \in N$, $P(n+1) \Rightarrow P(n)$ is true.
Which of the following is/are true?

(a) For any $n \in N$, $P(n)$.
(b) For any $n \in N$, $P(1) \Rightarrow P(n)$.
(c) For any $n \in N$, $(P(5) \wedge n < 5) \Rightarrow P(n)$.
(d) $P(10) \Rightarrow P(9)$.
(e) $P(9) \Rightarrow P(10)$.

42) Denote $((\text{not } p) \wedge q \wedge r) \vee (p \wedge (\text{not } q) \wedge (\text{not } r)) \vee (p \wedge q \wedge r)$ by α.
Which of the following is/are correct?

(a) α is false when p is true, q is true, r is false.
(b) α is false when p is true, q is false, r is false.
(c) α is false when p is false, q is false, r is true.
(d) α is false when p is false, q is true, r is false.
(e) α is false when p is true, q is true, r is true.

43) Let A = {0, 1} and the complement of 1 and 0 be 0 and 1 respectively. If A, +, * is to be a Boolean Algebra, what would be the values of x and y in the following definitions of + and * operations?

+	1	0
1	1	1
0	x	0

*	1	0
1	y	0
0	0	0

(a) x = 0, y = 0
(b) x = 0, y = 1
(c) x = 1, y = 0
(d) x = 1, y = 1
(e) x=0, y = 2

44) Let a, x and y be real numbers, u and v be positive numbers and $a > 0$ with $a \neq 1$. Which of the following is/are correct?

(a) $\log_2 2 = 1$.
(b) $\log_a (u/v) = \log_a (u - v)$.
(c) if $a^x = y$ then $x = \log_a y$.
(d) $\log_a 9 = 2\log_a 3$.
(e) $\log_a (uv) = \log_a u + \log_a v$.

45) $\dfrac{\sqrt{27a^3}}{\sqrt{3a}} =$

(a) $\sqrt{3}a^2$.
(b) $3a^2$.
(c) 3a.
(d) 9a.
(e) $\sqrt{3}a$.

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