UNIVERSITY OF COLOMBO, SRI LANKA

UNIVERSITY OF COLOMBO SCHOOL OF COMPUTING

DEGREE OF BACHELOR OF INFORMATION TECHNOLOGY (EXTERNAL)
Academic Year 2005/2006 – 1st Year Examination – Semester 1

IT1102: Mathematics for Computing-I

4th March 2006
(TWO HOURS)

Important Instructions :

- The duration of the paper is **2 (two) hours.**
- The medium of instruction and questions is English.
- The paper has **45 questions** and **9 pages.**
- All questions are of the MCQ (Multiple Choice Questions) type.
- All questions should be answered.
- Each question will have 5 (five) choices with **one or more** correct answers.
- All questions will carry equal marks.
- There will be a penalty for incorrect responses to discourage guessing.
- The mark given for a question will vary from –1 (*All the incorrect choices are marked & no correct choices marked*) to +1 (*All the correct choices are marked & no incorrect choices are marked*).
- Answers should be marked on the special answer sheet provided.
- Note that questions appear on both sides of the paper.
- If a page is not printed, please inform the supervisor immediately.
- Mark the correct choices on the question paper first and then transfer them to the given answer sheet which will be machine marked. **Please completely read and follow the instructions given on the other side of the answer sheet before you shade your correct choices.**

Notations:

Z – set of integers	N – set of positive integers
R – set of real numbers	ϕ - (null) empty set
S – Universal set	R^+- set of non-negative real numbers

1) Which of the following is(are) correct ?

(a) $\log_b 1 = 0$, for all b $\in$ N.

(b) $\log_b b = b$, for all b $\in$ N.

(c) $\log_b\left(\frac{u}{v}\right) = \frac{\log_b u}{\log_b v}$, for all u, v($\neq$0) $\in R^+$, b$\in$ N.

(d) $\log_b$ b = 1, for all b $\in$ N.

(e) $\log_b$ 1 = 1, for all b $\in$ N.

2) $(9^{1/2} \times 2^2) / (9^{3/2} \times 2^0) =$

(a) 0	(b) (4/9)	(c) 1
(d) (2/3)	(e) (3/2)	

3) Which of the following sets is(are) null (i.e. empty)?

(a) {x | x $\in$ Z and x > 0}	(b) {x | x $\in$ N and 15 < x < 16 }
(c) {x | x $\in$ Z and -16 < x < -15 }	(d) {x | x $\in$ Z and x < -3 }
(e) {x | x $\in$ N and x is prime, x is even}	

4) A = {2n | n$\in$N}, B = {3n | n$\in$N} and C = {4n | n$\in$N}. The only true statement is

(a) A $\cap$ C = C	(b) A $\cap$ B = ϕ	(c) A $\cup$ B = N
(d) A $\cup$ C = C	(e) A $\cup$ B $\cup$ C = A	

5) A = {2, 4, 6, 5, 7}, B = {6, 9, 8, 4} and C = {1, 2, 3, 6, 8, 12}.

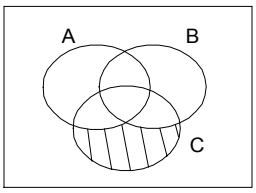
A $\cap$ (B $\cup$ C) equals

(a) {2, 4}	(b) {2, 4, 5, 6}	(c) {2, 4, 6}
(d) {1, 2, 4, 6}	(e) {1, 2, 6, 7}	

6) The number of different sets A satisfying the condition A $\cup$ {1, 2, 3, 4} = {1, 2, 3, 4, 5, 6} is

(a) 4	(b) 8	(c) 12
(d) 16	(e) 20	

7) Consider the following Venn diagram.



The shaded portion of the diagram represents

(a) $(A \cup B) \cap C$	(b) $A^c \cap B^c \cap C$	(c) $A \cap B \cap C$
(d) $A \cup (B \cap C)$	(e) $(A^c \cup B^c) \cap C$	

8) How many different ways are there for 4 boys and 3 girls to sit in a row if all the boys are to sit together and also all the girls are to sit together?

(a) 4! 3!	(b) 2. 4! 3!	(c) $\frac{4!3!}{2}$
(d) 7!	(e) $\frac{7!}{2}$	

9) Three girls A, B, C and three boys U, V, W go to a movie. In how many ways can they sit in a row if the girls are to sit together, and at the same time, A sits next to U?

(a) 4. 2! 2!	(b) 2. 2! 2!	(c) 6. 2! 2!
(d) 12. 2! 2!	(e) 8. 2! 2!	

10) How many different permutations can be formed out of all the letters of the word "MATHS"?

(a) 12	(b) 24	(c) 20
(d) 120	(e) 60	

11) How many different permutations can be formed out of the letters of the word "MISSISSIPPI"?

(a) (10!) / (7!)	(b) (10!) / (3!)	(c) (10!) / (2! 4! 4!)
(d) (10!) / (4!)	(e) (10!)/(4! 3!)	

12) In general ${}^nC_{(n-2)}$ is given by

(a) $n(n-1)(n-2)$	(b) $2n(n-1)$	(c) $\frac{n(n-2)}{2}$
(d) $\frac{n(n-1)}{2}$	(e) $\frac{n}{2}$	

13) ${}^0C_0$ is

(a) 0	(b) ∞	(c) not defined
(d) 1	(e) 0!	

14) The number of combinations of 5 objects taken 3 at a time will be equal to

(a) $\frac{5!}{3!}$	(b) $\frac{5!}{3!2!}$	(c) $\frac{5!}{2!}$
(d) 5.3!	(e) 5. 2!	

15) In how many ways can a committee consisting of 2 men and at least 2 women be chosen out of 4 men and 3 women?

(a) 4. $\binom{4}{2}$	(b) $\binom{4}{2}+\binom{3}{2}$	(c) $\binom{4}{2}.3$
(d) $\binom{7}{4}$	(e) 0	

16) Let A = {1,2} and B = {2,3}. Which of the following element(s) is/are in the power set P(A × B) ?

(a) ϕ	(b) (1,2)	(c) {(1,2)}
(d) (2,1)	(e) {(2,1)}	

17) If ${}^{(n+1)}C_r = 10$ and ${}^{n}C_{(r-1)} = 4$, then ${}^{n}C_r =$

(a) 6	(b) 14	(c) 40
(d) 28	(e) 210	

18) Ten balls numbered 0, 1, 2, ….., 9 are put in a box and two balls are taken at random. The probability that both numbers are odd is equal to

(a) $\frac{1}{9}$	(b) $\frac{2}{9}$	(c) $\frac{1}{3}$
(d) $\frac{4}{9}$	(e) 1	

19) If S is a finite equi-probable space and |A| denotes the number of elements in an event A, then for any arbitrary event E, P(A|E) is equal to

(a) $\frac{\lvert A\rvert}{\lvert E\rvert}$	(b) $\frac{\lvert A\cap E\rvert}{\lvert E\rvert}$	(c) $\frac{\lvert A\cap E\rvert}{\lvert S\rvert}$
(d) $\frac{\lvert A\rvert.\lvert E\rvert}{\lvert S\rvert}$	(e) |S|	

20) Let A and B be events with P(A) = $\frac{1}{4}$, P(B) = $\frac{1}{3}$, and P(A $\cup$ B) = $\frac{1}{2}$. Then P(B|A^c) is equal to

(a) $\frac{1}{4}$	(b) $\frac{3}{4}$	(c) $\frac{1}{3}$
(d) $\frac{1}{12}$	(e) 0	

21) A box contains 4 red balls and 2 white balls. Another box contains 5 red balls and 3 white balls. A box is selected at random, and then a ball is drawn at random. The probability that the ball is white is equal to

(a) $\frac{1}{6}$	(b) $\frac{3}{16}$	(c) $\frac{1}{6} \cdot \frac{3}{16}$
(d) $\frac{1}{6} + \frac{3}{16}$	(e) 0	

22) We are given 2 coins of which one is a fair coin and the other has Heads on both sides. A coin is selected at random and tossed. The probability of getting Tails is equal to

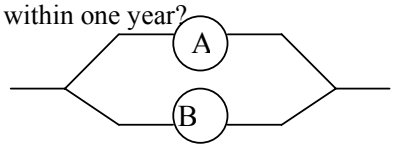
(a) 0	(b) $\frac{1}{2}$	(c) $\frac{1}{4}$
(d) $\frac{3}{4}$	(e) 1	

23) The probabilities that each of 2 shooters hit the target are respectively $\frac{1}{2}$ and $\frac{2}{3}$.

The probability that exactly one man hits the target is equal to

(a) $\frac{1}{6}$	(b) $\frac{1}{3}$	(c) $\frac{1}{2}$
(d) 1	(e) 0	

24) The system shown in the following diagram has 2 components A and B attached to it. The failure or non failure of each component is independent of the other and each has probability 0.1 of failing within 1 year. The system works as long as at least one side works. What is the probability that the system does not fail within one year?



(a) 0.9	(b) 0.99	(c) 0.81
(d) 0.01	(e) 0	

25) A table tennis player has probability $\frac{1}{2}$ of winning (W) and $\frac{1}{2}$ of losing (L). Find the probability that he wins a series of 3 games. (Note that if he wins the first 2, he wins the series and the game stops. Otherwise he has to win 2 out of 3).

(a) $\frac{3}{8}$	(b) $\frac{1}{4}$	(c) $\frac{1}{8}$
(d) $\frac{1}{2}$	(e) 1	

26) Let S = {1,2,3,4,5} and T be the relation {(1,3),(2,4),(3,5),(1,1),(2,2),(4,2),(3,1)} on S. Which of the following is (are) correct?

(a) T is a reflexive relation.	(b) T is a symmetric relation.
(c) T is a transitive relation.	(d) T is not a reflexive relation.
(e) T is not a transitive relation.	

27) Let A = {a, b, c}, B = {1, 2} and the function f : A $\rightarrow$ B be defined as f(a) = 1, f(b) = 2, f(c) = 2. Which of the following is (are) correct?

(a) f is an onto function.	(b) f is a 1-1 function.
(c) f is not an onto function.	(d) f is not a 1-1 function.
(e) f is a bijection	

28) Consider the following truth tables.

(i)

p	q	$p \Rightarrow (p \Rightarrow q)$
F	T	T
T	F	F
T	T	T
F	F	T

(ii)

p	q	$p \vee$ not q
T	F	T
T	T	T
F	F	T
F	T	F

(iii)

p	q	$p \Rightarrow (p \wedge q)$
F	F	T
T	T	T
T	F	F
F	T	T

Which of the following are true statements?

(a) (i), (ii) and (iii) are all correct.	(b) Only (i) is correct.
(c) (i), (iii) are correct but (ii) is incorrect.	(d) (ii), (iii) are correct but (i) is incorrect.
(e) (i), (ii) and (iii) are all incorrect.	

29) Consider the following truth table.

p	q	r	φ_1	φ_2	φ_3
T	T	T	T	T	T
T	F	T	T	T	T
F	T	T	T	T	F
F	F	T	F	T	F
T	T	F	T	T	T
T	F	F	T	F	F
F	T	F	F	F	F
F	F	F	F	F	F

Now, $p \wedge (q \vee r)$ is one of φ_1, φ_2, φ_3, and $p \vee (q \wedge r)$ is also one of φ_1, φ_2, φ_3. Which of the following is(are) correct?

(a) $p \wedge (q \vee r)$ is φ_2 and $p \vee (q \wedge r)$ is φ_3	(b) $p \wedge (q \vee r)$ is φ_1 and $p \vee (q \wedge r)$ is φ_2
(c) $p \wedge (q \vee r)$ is φ_3 and $p \vee (q \wedge r)$ is φ_2	(d) $p \wedge (q \vee r)$ is φ_2 and $p \vee (q \wedge r)$ is φ_1
(e) $p \wedge (q \vee r)$ is φ_3 and $p \vee (q \wedge r)$ is φ_1	

30) Denote $((\text{not } p) \wedge q \wedge r) \vee (p \wedge (\text{not } q) \wedge (\text{not } r)) \vee (p \wedge q \wedge r)$ by φ_1.

Which of the following are(is) correct?

(a) φ_1 is false when p is true, q is true, r is false.	(b) φ_1 is false when p is true, q is false, r is false.
(c) φ_1 is false when p is false, q is false, r is true.	(d) φ_1 is false when p is false, q is true, r is false.
(e) φ_1 is false when p is true, q is true, r is true.	

31) Out of the 8 possible sets of truth values for p, q, r,
$(p \vee (\text{not } q) \vee (\text{not } r)) \wedge (p \vee q \vee r) \wedge (p \vee (\text{not } q) \vee r)$ is <u>true only for</u>:

(a) 6 sets of values.	(b) 5 sets of values.	(c) 4 sets of values.
(d) 3 sets of values.	(e) 2 sets of values.	

32) Consider the following:
(i) $(p \vee (\text{not } p)) \Rightarrow (p \wedge (\text{not } p))$.
(ii) $(p \wedge (\text{not } q)) \Leftrightarrow (q \vee (\text{not } p))$
(iii) $p \vee (q \wedge (\text{not } p)) \vee ((\text{not } q) \wedge (\text{not } p))$.

Which of the following are(is) correct?

(a) (i) is a contradiction.	(b) (iii) is a tautology.
(c) (ii) is neither a contradiction nor a tautology.	(d) (iii) is neither a contradiction nor a tautology.
(e) (ii) is a contradiction.	

33) It is given that $p \vee q \vee r$, $p \Rightarrow r$ and $q \Rightarrow s$ are true. Then which of the following must necessarily be true?

(a) $p \vee q$	(b) $p \vee r$	(c) $r \vee s$
(d) $p \vee r \vee s$	(e) $q \vee s$	

34) Let $S = \{1, 2, 3\}$ be the universal set and $x,y \in R$. Now consider the following:
(i) $\exists x \, \forall y, x^2 < y + 1$ (ii) $\forall x \, \exists y, x^2 + y^2 < 12$ (iii) $\forall x \, \forall y, x^2 + y^2 < 12$

Which of the following is/ are correct?

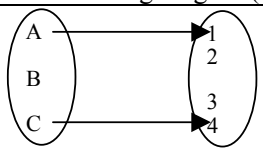
(a) (i), (ii) are true but (iii) is false.	(b) (i) is true but (ii), (iii) are false.
(c) (ii), (iii) are true but (i) is false.	(d) (i), (ii), (iii) are all false.
(e) (i), (ii), (iii) are all true.	

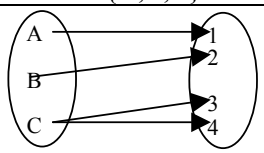
35) For $n \in N$, P(n) is a proposition such that for any $n \in N$, $P(n) \Rightarrow P(n + 2)$ is true.

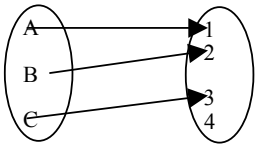
Which of the following is(are) true?

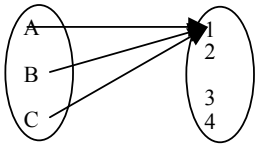
(a) For any $n \in N$, $P(1) \Rightarrow P(n)$.	(b) For any $n \in N$, $(P(1) \vee P(2)) \Rightarrow P(n)$.
(c) For any $n \in N$, P(n).	(d) For any $n \in N$, $(P(1) \wedge P(2)) \Rightarrow P(n)$.
(e) For any $n \in N$, $P(2) \Rightarrow P(n)$.	

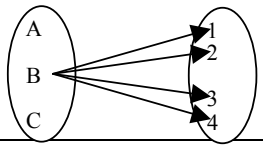
36) Which of the following diagram(s) define(s) a function(s) from X = {A,B,C} to Y = {1,2,3,4}?

(a) 

(b) 

(c) 

(d) 

(e) 

37) Let $\sum = \{a,b\}$ and $\sum^*$ be the set of all strings over $\sum$. The function g: $\sum^* \rightarrow Z$ is defined as g(s) = the number of a's in s, for each string $s \in \sum^*$. Then g(ababb) =

(a) 0	(b) 2	(c) 3
(d) 4	(e) 5	

38) Let $\sum = \{0,1\}$ and $\sum^n$ be the set of all strings of length n over $\sum$. The function h: $\sum^n \times \sum^n \rightarrow Z$ is defined as h(s,t) = the number of positions in which s and t have different values, for each pair (s,t) $\in \sum^n \times \sum^n$. Which of the following is(are) true ?

(a) h(1,1) = 0	(b) h(10,01) = 2	(c) h(1,1) = 1
(d) h(10,01) = 0	(e) h(10,01) = 1	

39) Consider the following :

(i) 4 + 2 = 8 (ii) 4 + 2 = 6

(iii) $x^2 = 4$ (iv) $x^2 = -1$

(v) For any real number x, $x^2 = -1$.

Which of the following is (are) correct ?

(a) (i) is a proposition.	(b) (ii) is a proposition.
(c) (iii) is a proposition.	(d) (iv) is a proposition.
(e) (v) is a proposition.	

40) Let S = {2,5,7,9,12} be the universal set and $x \in S$.

Now, consider the following.

(i) $\forall x,\ 1 < x < 9$ (ii) $\exists x,\ x^2 = 25$ (iii) $\forall x,\ -4 < x - 4 < 9$

(a) (i) is true.	(b) (ii) is true.	(c) (iii) is true.
(d) (i) is false.	(e) (iii) is false.	

41) Let f: S $\rightarrow$ T, g : T $\rightarrow$ U and h: U $\rightarrow$ V be functions. Then which of the following is (are) true?

(a) (h . g) . f = h . (g . f)	(b) (h . g) . f $\neq$ h . (g . f)
(c) if f and g are 1-1 then g .f is 1-1	(d) if f and g are 1-1 then g .f is not 1-1
(e) if f and g are onto then g .f is onto.	

42) Let A = {1,2} and B = {3,7} and the function f: A $\rightarrow$ B be defined as f(1) = 3, f(2) = 7. Then which of the following is (are) correct?

(a) $f^{-1}(3) = 1$	(b) $f^{-1}(7) = 2$	(c) $f^{-1}(7) = 1$
(d) $f^{-1}(3) = 2$	(e) $f^{-1}(1) = 1$	

43) A, B are subsets of a universal set S, A is a subset of B and A $\neq$ B. Which of the following statements are(is) true?

(a) $B^c \cap A = \phi$.	(b) $A \cap B \neq B$.	(c) $A \cap A^c = \phi$.
(d) $A \cap B = A$	(e) B is a subset of $A \cap B$.	

Questions 44) and 45) are based on the following:

Let <B,+,*,',0,1> be a Boolean Algebra, where B is a set , + and * are the sum and product operations respectively defined on the elements of B, 0 and 1 are the zero and unit elements respectively, ' is the complement (negation) operator and a and b are elements in B.

44) Which of the following is(are) correct?

(a) a + 0 = 0 + a = 0	(b) a + 0 = 0 + a = a	(c) a *1 = 1 * a = a
(d) a *1 = 1 * a = 1	(e) a + (a + b) = a + b	

45) Which of the following is(are) correct?

(a) a + b = b + a	(b) a + b = a * b	(c) a + 0 = a
(d)a + 0 = 0	(e) a + 0 = 1	

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