UNIVERSITY OF COLOMBO, SRI LANKA

UNIVERSITY OF COLOMBO SCHOOL OF COMPUTING

DEGREE OF BACHELOR OF INFORMATION TECHNOLOGY (EXTERNAL)

Academic Year 2011 /2012 – 1st Year Examination – Semester 2

IT2104 - Mathematics for Computing I

28th July 2012

(TWO HOURS)

Important Instructions :

- The duration of the paper is 2 (two) hours.
- The medium of instruction and questions is English.
- The paper has **45** questions and **11** pages.
- All questions are of the MCQ (Multiple Choice Questions) type.
- All questions should be answered.
- Each question will have 5 (five) choices with **one or more** correct answers.
- All questions will carry equal marks.
- There will be a penalty for incorrect responses to discourage guessing.
- The mark given for a question will vary from 0 (*no correct choices are marked*) to +1 (*All the correct choices are marked & no incorrect choices are marked*).
- Answers should be marked on the special answer sheet provided.
- Note that questions appear on both sides of the paper.
- If a page is not printed, please inform the supervisor immediately.
- Mark the correct choices on the question paper first and then transfer them to the given answer sheet which will be machine marked. **Please completely read and follow the instructions given on the other side of the answer sheet before you shade your correct choices.**

Notations:

Z – set of integers	N – set of positive integers
R – set of real numbers	∅ - (null) empty set
U – Universal set	R^+ - set of positive real numbers

1) $\left(\frac{27x^3}{8a^{-3}}\right)^{\frac{-2}{3}}$ is equal to

(a) $\left(\frac{3xa}{2}\right)^{-2}$ (b) $\left(\frac{2}{3xa}\right)^{-2}$ (c) $\left(\frac{3xa}{2}\right)^{2}$ (d) $\frac{2}{3xa}$ (e) $\left(\frac{2}{3xa}\right)^{2}$

2) $\ln \frac{(ab^2c^4)^{\frac{1}{6}}}{(a^{-3}b^3c^6)^{\frac{1}{9}}}$ where $a, b, c \in R^+$ is equal to

(a) $\ln \sqrt{a}$ (b) $\ln \frac{\sqrt{a}}{bc}$ (c) $\ln \left(\frac{a^4}{bc^2}\right)^{\frac{1}{18}}$ (d) $\frac{1}{2} \ln a$ (e) $\ln \left(\frac{a^2}{bc}\right)^{\frac{1}{9}}$

3) $5\log_2 6 - (\log_2 4 + \log_2 243)$ is equal to

(a) $\log_2 81$ (b) $\log_2 27$ (c) 3 (d) $\log_2 241$ (e) 0

4) Let $A = \{(a,b) \mid a,b \in N \text{ and } a^2 + b^2 = 17\}$ and $B = \{(a,b) \mid a,b \in Z \text{ and } a - b = 5\}$. Then $A \cap B$ equals

(a) {(4,1)} (b) {(1,4)} (c) {(4,-1)} (d) {(1,4), (4,-1)} (e) ∅

5) Let A and B be two non-empty sets. Which of the following is/are true?

(a) $A \cup B \subseteq A$ (b) $A \subseteq A$ (c) $A \subseteq A \cap B$ (d) $U^c = \varnothing$ (e) $A \subseteq \varnothing^c$

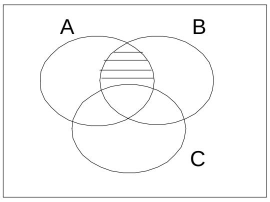
6) Let A be any non-empty set. If P(A) is the power set of A, which of the following is/are true?

(a) $A \in P(A)$.	(b) $\varnothing \in P(A)$	(c) $\lvert P(A)\rvert = 2^{\lvert A\rvert}$.	(d) $P(A) \cap A = P(A)$.
(e) $P(A) \cup A = P(A)$			

7) Let P, Q, R and A be non-empty sets such that $A = P \cup (Q \cap R)$. Which of the following is/are true?

(a) $A^c = P^c \cap (Q^c \cap R^c)$	(b) $A^c = P^c \cap (Q \cap R)^c$	(c) $A^c = P^c \cap (Q^c \cup R^c)$
(d) $A^c = P \cap (Q \cup R)$	(e) $A^c = ((A \setminus P) \setminus Q) \setminus R$	

8) Consider the following Venn diagram.



Which of the following sets is represented by the shaded portion?

(a) $(A \cap B)^c \setminus (A \cap B \cap C)$	(b) $A^c \cap (B \cup A)$.
(c) $(A \cap B \cap C)^c \setminus (A \cap B)^c$.	(d) $(B \cup C)^c \cap A$.
(e) $(A \cap B) \setminus (A \cap B \cap C)$.	

9) Let A be a non empty set. Which of the following is(are) correct?

(a) $(A \cup B) \cap (A \cup C) = A \cap (B \cup C)$	(b) $A \cup (B \cup C) = (A \cup B) \cup C$.
(c) $A \cup (B \cap C) = A \cap (B \cup C)$	(d) $A \cap (B \cap C) = (A \cap B) \cap C$.
(e) $(A \cap B) \cup (A \cap C) = A \cap (B \cup C)$	

10) Let A,B and C be any three sets. Which of the following are propositions?

(a) $(A \subseteq B \wedge B \subseteq A) \rightarrow A = B$.	(b) $A \subseteq B \wedge B \subseteq C$.
(c) $(A \subset B \wedge B \subset C) \rightarrow A \subset C$.	(d) $A \subseteq \varnothing$.
(e) Check whether you have correctly shaded your index number.	

11) Let *p* and *q* be two atomic propositions. Which of the following is(are) contradiction(s) ?

(a) $(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$.
(b) $p \vee (q \vee \sim p)$.
(c) $\sim((p \rightarrow q) \leftrightarrow (\sim p \vee q))$.
(d) $p \wedge (q \wedge \sim q)$.
(e) $(p \rightarrow q) \leftrightarrow (\sim p \vee q)$.

12) Consider the following truth table of the proposition Φ with three propositional variables *p*, *q* and *r*.

p	q	r	Φ
T	T	T	F
T	T	F	T
T	F	T	F
F	T	T	F
T	F	F	F
F	T	F	F
F	F	T	F
F	F	F	F

Which of the following could be Φ?

(a) $(p \wedge q \Rightarrow p \vee q) \vee \sim r$.
(b) $p \wedge \sim(\sim q \vee r)$.
(c) $(p \wedge q \wedge \sim r) \vee (\sim p \wedge q \wedge r) \vee (\sim p \wedge \sim q \wedge r)$.
(d) $p \wedge q \wedge \sim r$.
(e) $(\sim p \vee \sim q \vee r) \wedge (p \vee \sim q \vee \sim r) \wedge (\sim p \vee \sim q \vee r)$.

13) Which of the following pairs of propositions is(are) equivalent?

(a) $p \vee q$, $p \wedge q$.
(b) $p \wedge (p \vee q)$, p.
(c) $p \wedge (\sim p \vee q)$, $p \wedge q$.
(d) $p \vee (p \wedge q)$, p.
(e) $(p \rightarrow q)$, $(\sim p \rightarrow \sim q)$.

14) There are four children Nadili, Sachini, Yoshitha and Sanka. Three of these children always tell lies and one of them always tells the truth. A vase was broken by one of the children. This is what they said.

Nadili: Sanka broke the vase.
Sanka: Yoshitha broke the vase.
Sachini: I did not break the vase.
Yoshitha: Sanka lied when he said he broke the vase.

Which of the following is(are) true?

(a) Sanka broke the vase.
(b) Yoshitha broke the vase.
(c) Sachini broke the vase.
(d) Nadili broke the vase.
(e) Nadili did not break the vase.

15) Which set(s) of the following statements is(are) inconsistent?

(a) $p \wedge q$, $\sim p$, q	(b) $p \vee q$, p, $\sim q$	(c) $p \to r$, p, $\sim r$
(d) $(q \leftrightarrow p)$, $\sim p$, q	(e) $\sim q \Rightarrow p$, $p \Rightarrow \sim r$, r	

16) Which of the following arguments is(are) a fallacy(fallacies)?

(a) p, $(p \Rightarrow q) \vdash q$	(b) $(p \Rightarrow q)$, $q \vdash p$	(c) $(p \Rightarrow \sim q)$, $(p \vee r)$, $(r \Rightarrow \sim s)$, $s \vdash q$
(d) p, $(\sim p \vee q) \vdash q$	(e) $(p \Rightarrow q)$, $\sim q \vdash \sim p$	

17) Let $p(x)$ and $q(x)$ be two predicates of the variable x defined by $x < 1$ and $x > 2$ respectively where $x \in N$. Which of the following propositions is(are) true?

(a) $\exists x\, p(x) \vee \exists x\, q(x)$	(b) $\exists x\, (p(x) \vee q(x))$	(c) $\exists x\, p(x)$
(d) $\exists x\, p(x) \wedge \exists x \sim q(x)$	(e) $\exists x\, (p(x) \wedge q(x))$	

18) Let $p(x)$ and $q(x)$ be two predicates of the variable x. Consider the following propositions.

(i) $\forall x\, (\sim p(x) \vee q(x))$
(ii) $\exists x\, (p(x) \wedge \sim q(x))$
(iii) $\forall x\, (p(x) \wedge q(x))$
(iv) $\sim \exists x \sim (p(x) \wedge q(x))$
(v) $\sim \forall x\, (\sim p(x) \vee q(x))$
(vi) $\sim \exists x\, (p(x) \wedge \sim q(x))$

Identify the equivalent proposition pairs from among the above.

(a) (i) and (vi)	(b) (i) and (iv)	(c) (ii) and (v)
(d) (iii) and (iv)	(e) (ii) and (iii)	

19) Suppose x, y$\in$ {5, 10, 15, 20, 25}. Which of the following propositions is(are) true?

(a) $\forall x\ \exists y\ x \leq y$.	(b) $\forall y\ \exists x\ y < x$.	(c) $\exists x\ \forall y\ x \leq y$.
(d) $\exists x\ \exists y\ x < y$.	(e) $\forall x\ \forall y\ x \leq y$.	

20) Let p(x) be a predicate of the variable x defined in a non-null set D. Which of the following must be true if the proposition $\forall x\ p(x)$ is false?

(a) There is an x in D for which p(x) is false.
(b) For every x in D, p(x) is false
(c) For every x in D, ~p(x) is false.
(d) There are no elements in D for which p(x) is true.
(e) There are x_0 and x_1 in D for which p(x) is false.

21) Let p and q be two atomic propositions. Which of the following propositions is(are) expressed in Canonical Conjunctive Normal Form?

(a) $(p \vee q \vee r) \wedge (p \vee \sim q \vee r) \wedge (p \vee \sim q \vee r)$.
(b) $(p \wedge q \wedge r) \vee (p \wedge \sim q \wedge r) \vee (q \rightarrow p \wedge r)$.
(c) $(p \vee q \vee r) \wedge (p \vee \sim q \vee r) \wedge (q \rightarrow p \vee r)$.
(d) $p \wedge (p \vee \sim q \vee r) \wedge (p \vee \sim q \vee r)$.
(e) $(p \wedge q \wedge r) \vee (p \wedge \sim q \wedge r) \vee (\sim p \wedge q \wedge r)$.

22) Let p(x) and q(x) be predicates of variable x defined on A = {a}.

Which of the following propositions is(are) true if $\exists x\ (p(x) \vee q(x))$ is false ?

(a) $\sim p(a) \vee \sim q(a)$.	(b) $p(a) \wedge q(a)$	(c) $\sim p(a) \wedge q(a)$
(d) $\sim p(a) \wedge \sim q(a)$	(e) $\forall x \sim(p(x) \vee q(x))$.	

23) Let A, B and C be three distict sets. If A × B denotes the Cartesian product of A and B, which of the following is/are true ?

(a) $A \times B = B \times A$.
(b) $A \times \emptyset = \emptyset \times A$.
(c) $A \times (B \cap C) = (A \times B) \cap (A \times C)$.
(d) $A \times (B \cup C) = A \times (B \cap C)$.
(e) $A \times (B \cup C) = (A \times B) \cup (A \times C)$.

24) Suppose a non-empty relation ρ is defined in a non-null set X. Then ρ is said to be reflexive if

(a) $\forall x, x \in D(\rho) \Rightarrow (x,x) \in \rho$	(b) $\exists x, x \in D(\rho) \Rightarrow (x,x) \in \rho$
(c) $\forall x, x \in D(\rho) \wedge (x,x) \in \rho$	(d) $\exists x, x \in D(\rho) \wedge (x,x) \in \rho$
(e) $\forall x, x \in D(\rho) \vee (x,x) \in \rho$	

25) Let A={a,b,c} and β ={ (a,a), (b,b), (a,b), (b,a), (c,a), (a,c) } be a relation in A.

Which of the following is(are) true?

(a) β is symmetric.	(b) β is reflexive.
(c) β is transitive.	(d) β is not symmetric.
(e) β is not transitive.	

26) Let α and ρ be two relations defined in a set given by
α={(5,6),(7,9),(8,3),(4,4)} and β={(6,1),(9,9),(8,5),(6,12),(10,4)}.

Then $\beta o \alpha$ equals

(a) {(5,1),(5,12),(7,9)}
(b) {(8,6),(10,4)}
(c) {(5,6),(7,9),(8,3),(4,4),(6,1),(9,9),(8,5),(6,12),(10,4)}
(d) $\{ (x,y) \mid \exists z\ (x,z)\ \varepsilon\ \alpha \wedge (z,y)\ \varepsilon\ \beta \}$
(e) {(8,6)}

27) Suppose ρ is the equivalence relation defined in A={1,2,3,4} by
ρ = { (1,1), (2,2), (3,3), (4,4), (1,2), (2,1), (2,3), (3,2), (1,3), (3,1)}.

$[3]_\rho \cup [4]_\rho$ equals

(a) {1,2,3}.	(b) {4}.	(c) {1,2}.
(d) A.	(e) D(ρ).	

28) Let A be a non-empty subset of N and let the two relations α and β be defined in A by
$\alpha=\{(x,y) \mid x,y \in A, x = y\}$ and $\beta=\{(x,y) \mid x,y \in A, x < y\}$.

Which of the following is(are) true?

(a) α is Reflexive and β is Symmetric.	(b) α is Reflexive and β is Transitive
(c) α and β are both Symmetric .	(d) α and β are both Transitive.
(e) α and β are both Reflexive.	

29) Suppose ρ is a relation defined in a non-null set. Which of the following is(are) true?

(a) $D(\rho)=\{ y \mid \exists x\ (y,x)\in \rho\}$, .
(b) $R(\rho)=\{ x \mid \exists y\ (y,x)\in \rho\}$.
(c) $D(\rho)=\{ x \mid \exists y\ (y,x)\in \rho\}$.
(d) $R(\rho)=\{ y \mid \exists x\ (y,x)\in \rho\}$.
(e) $R(\rho)=\{ y \mid \exists x\ (x,y)\in \rho\}$..

30) Let f be a **1-1** function and $x_1,x_2\in D(f)$. Which of the following is(are) true?

(a) $\sim(\exists x_1, \exists x_2\ , x_1 \neq x_2 \ \wedge\ f(x_1) = f(x_2))$.
(b) $\sim(\exists x_1, \exists x_2\ , x_1 \neq x_2 \ \vee\ f(x_1) = f(x_2))$.
(c) $\forall x_1, \forall x_2\ \ x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$.
(d) $\forall x_1, \forall x_2\ \ x_1 = x_2 \vee f(x_1) \neq f(x_2)$.
(e) $\forall x_1, \forall x_2\ \ x_1 = x_2 \wedge f(x_1) \neq f(x_2)$.

31) Suppose f is a 1-1 function.
Which of the following is/(are) correct about f^{-1} if $D(f^{-1}) = R(f^{-1})$?

(a) f^{-1} is 1-1.
(b) $D(f^{-1})=D(f)$.
(c) $R(f^{-1})=R(f)$.
(d) $f^{-1}\ o\ f$ is 1-1 .
(e) $f\ o\ f^{-1}$ is 1-1.

32) Let A={a,b} and B={x,y}. Which of the following pairs represent(s) the total number of **into** functions and **onto** functions respectively from A to B?

(a) (2,2).
(b) (4,4).
(c) (4,2).
(d) (6,2).
(e) (2,4).

33) Let the functions f and g be defined on R as $f(x) = 2x-1$ and $g(x) = x+1$. Then g ***o*** f isx defined by

(a) $D(g\ o\ f) = R,\ (g\ o\ f)(x) = 2x+3$.
(b) $D(g\ o\ f) = R,\ (g\ o\ f)(x) = x+3$.
(c) $D(g\ o\ f) = R,\ (g\ o\ f)(x) = 2x$.
(d) $D(g\ o\ f) = R,\ (g\ o\ f)(x) = 2x+1$.
(e) $D(g\ o\ f) = R,\ (g\ o\ f)(x) = 2x -1$.

34) Consider the function f defined by $D(f)=\{x|\ x \in R, x \neq 3\}$, $f(x)=(x+2)/(x-3)$. Which of the following define(s) f^{-1}?

(a) $D(f^{-1})=\{x|\ x \in R, x \neq 2\}$ and $f^{-1}(x)=(x+3)/(x-2)$.
(b) $D(f^{-1})=\{x|\ x \in R, x \neq 1\}$ and $f^{-1}(x)=(3x+2)/(x-1)$.
(c) $D(f^{-1})=\{x|\ x \in R, x \neq 1\}$ and $f^{-1}(x)=(3x-2)/(x-1)$.
(d) $D(f^{-1})=\{x|\ x \in R, x \neq 2\}$ and $f^{-1}(x)=(x-3)/(x-2)$.
(e) $D(f^{-1})=\{x|\ x \in R, x \neq 2\}$ and $f^{-1}(x)=(2x+3)/(x-2)$.

35) By using a letter in the word "COMPUTER" not more than once, how many three letter words can be formed if they should end with R?

(a) 1.	(b) 48.	(c) 42.
(d) 336.	(e) 8.	

36) How many ways are there for four Cricketers, three Swimmers and two Athletes be stand in a line if the same type of sports persons stand together?

(a) 432	(b) 48	(c) 1728
(d) 24	(e) 362880	

37) A committee of 5 is to be selected from 7 men and 5 women. In how many ways can this committee be formed if no more than 2 women are to be included?

(a) 175	(b) ${}^7C_5 + {}^7C_4 * {}^5C_1 + {}^7C_3 * {}^5C_2$	(c) 350
(d) 546	(e) ${}^7P_5 + {}^7P_4 * {}^5P_1 + {}^7P_3 * {}^5P_2$	

38) Let $(A, \cup, \cap)$ be a Boolean algebra with A representing the power set of a non-empty set X. If $P \in A$, which of the following state(s) the identity laws of this Boolean algebra ?

(a) $P \cup P^c = A$, $P \cap P^c = \varnothing$.	(b) $P \cup \varnothing = P$, $P \cap X = P$.	(c) $P \cap \varnothing = \varnothing$, $P \cup X = X$.
(d) $\varnothing^c = X$, $X^c = \varnothing$.	(e) $P \cup P = P$, $P \cap P = P$.	

39) Let E and F be two disjoint events and P (E) =P (F) =1/3. The probability that neither event will occur is:

(a) $P(E \cup F) = P(E) + P(F) = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$
(b) $P(E \cap F) = P(E).P(F) = \frac{1}{3}.\frac{1}{3} = \frac{1}{9}$
(c) $P(\bar{E} \cap \bar{F}) = P(\bar{E}).P(\bar{F}) = \frac{2}{3}.\frac{2}{3} = \frac{4}{9}$
(d) $P(\overline{(E \cup F)}) = 1 - P(E) - P(F) = 1 - \frac{1}{3} - \frac{1}{3} = \frac{1}{3}$
(e) $P(E \cup F) = P(E) + P(F) - P(E \cap F) = \frac{1}{3} + \frac{1}{3} - \frac{1}{9} = \frac{5}{9}$

40) Let a sample space be $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5\}$ where the sample points have equal probability,

$$P(\{\omega_1\}) = P(\{\omega_2\}) = P(\{\omega_3\}) = P(\{\omega_4\}) = P(\{\omega_5\}) = \frac{1}{5}$$

Then,

(a) the events $E = \{\omega_1, \omega_2\}$ *and* $H = \{\omega_2\}$ are independent.
(b) the events $E = \{\omega_1, \omega_2\}$ *and* $G = \{\omega_2, \omega_3\}$ are independent.
(c) the events $E = \{\omega_1, \omega_2\}$ *and* Ω are not independent.
(d) the events $E = \{\omega_1, \omega_2\}$ *and* $F = \{\omega_3, \omega_4\}$ are not independent.
(e) the events $E = \{\omega_1, \omega_2\}$ *and* $I = \{\omega_2, \omega_3, \omega_4\}$ are independent.

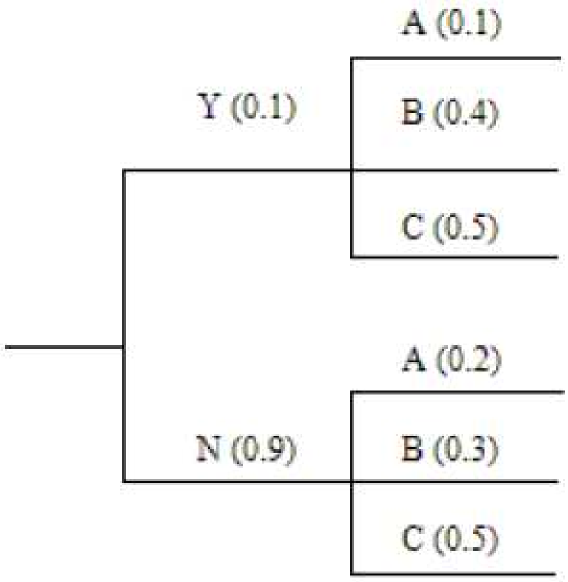
41) The probability that a research project will be well planned is 0.80.Furthermore, the probability that the project will be well planned and successful is 0.72. What is the probability that the research project will be successful given that it was well planned?

(a) 0.08	(b) 0.57	(c) 0.72	(d) 0.80	(e) 0.90

42) The General Social Survey asked subjects whether they favored or opposed the death penalty for persons convicted of murder (F = favor, O = oppose) and whether they favored or opposed a law which would require a person to obtain a permit before he or she could buy a gun (1 = favor, 2 = oppose) as well as if they had ever been married (Y = yes, N = no). If a tree diagram is used to list the sample space with the first branch representing the response to the death penalty question, the second the response to the gun permit question and the third the response to the marriage question, which of the following is a possible outcome?

(a) N2F	(b) F1Y	(c) FOY	(d) 1ON	(e) OON

43) Suppose the following tree diagram summarizes the responses of 500 people to two questions, where the first response is either yes or no and the second question is a multiple choice question with three possible answers (A, B or C). Use the tree diagram to calculate the probability that a person answered B to the multiple choice question.



(a) 0.7	(b) 0.04	(c) 0.07	(d) 0.4	(e) 0.31

44) Which one of the following statements is true?

(a) For any event A, if $A \cup \bar{A} = S$ then $A \cap \bar{A} = \emptyset$.
(b) For any two events A and B $\overline{A \cup B} = \bar{A} \cup \bar{B}$.
(c) An event is not a subset of a sample space.
(d) A probability is a real number which is less than 1.
(e) $P(A \cup B) = P(A) + P(B)$.

45) Let E and F be two events such that $E \subseteq F$ .The probability of the event F can be expressed as the sum of the probabilities of two disjoint events as follows.

(a) $P(F) = P(E) + P(F \cap \bar{E})$
(b) $P(F) = P(E \cup F) - P(E \cap F)$
(c) $P(F) = P(E) + P(E \cap \bar{F})$
(d) $P(F) = P(E \cup \bar{F}) - P(\bar{E} \cap F)$
(e) $P(F) = P(\bar{E}) - P(E)$

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