UNIVERSITY OF COLOMBO, SRI LANKA

UNIVERSITY OF COLOMBO SCHOOL OF COMPUTING

DEGREE OF BACHELOR OF INFORMATION TECHNOLOGY (EXTERNAL)

Academic Year 2012 /2013 – 1st Year Examination – Semester 2

IT2104 - Mathematics for Computing I

27th July 2013

(TWO HOURS)

Important Instructions :

- The duration of the paper is 2 (two) hours.
- The medium of instruction and questions is English.
- The paper has **43** questions and **11** pages.
- All questions are of the MCQ (Multiple Choice Questions) type.
- All questions should be answered.
- Each question will have 5 (five) choices with **one or more** correct answers.
- All questions will carry equal marks.
- There will be a penalty for incorrect responses to discourage guessing.
- The mark given for a question will vary from 0 (*no correct choices are marked*) to +1 (*All the correct choices are marked & no incorrect choices are marked*).
- Answers should be marked on the special answer sheet provided.
- Note that questions appear on both sides of the paper.
- If a page is not printed, please inform the supervisor immediately.
- Mark the correct choices on the question paper first and then transfer them to the given answer sheet which will be machine marked. **Please completely read and follow the instructions given on the other side of the answer sheet before you shade your correct choices.**

Notations:

Z – set of integers N – set of positive integers
R – set of real numbers ∅ - (null) empty set
U – Universal set R^+ - set of positive real numbers

1) $\frac{27x^{27}}{64y^{63}}$ is equal to

(a) $\left(\frac{3x^3}{4y^{21}}\right)^3$. (b) $\left(\frac{3x^9}{4y^{21}}\right)^3$. (c) $\left(\frac{9x^9}{18y^{21}}\right)^3$. (d) $\left(\frac{3x^3}{4y^7}\right)^3$. (e) $\left(\frac{3y^{-21}}{4x^{-9}}\right)^3$.

2) $-3 + \log_2 120 - \log_2 5$ is equal to

(a) $\log_2 8$. (b) $\log_2 3$. (c) $(\log_3 2)^{-1}$. (d) $\log_2 10 * \log_{10} 3$.
(e) $\log_{10} 2 * \log_3 10$.

3) Which of the following is(are) correct?

(a) $\forall a \in N, \forall u, v \in R^+ \ \log_a\left(\frac{u}{v}\right) = \log_a u - \log_a v$.
(b) $\forall a \in N, \forall u, v \in R \ \log_a\left(\frac{u}{v}\right) = \log_a u - \log_a v$.
(c) $\forall a \in N, \log_a 1 = 0$.
(d) $\forall a \in Z, \log_a 1 = 0$.
(e) $\forall a, u, v \in N \ \log_a\left(\frac{u}{v}\right) = \log_a u - \log_a v$.

4) Let $A = \{1,3,5,7\}$ and $B = \{2,4,6\}$. Then, which of the following is(are) correct?

(a) $A \cup B = \{x | x \in N \wedge x < 8\}$. (b) $A \setminus B = A$. (c) $A \cup B = \emptyset$.
(d) $A \cap B = \{1,2,3,4,5,6,7\}$. (e) $A \cap B = \emptyset$.

5) Let A and B be two non-empty sets. Which of the following is/are true?

(a) $A \cap B \subseteq A \cup B$. (b) $A \subseteq A$. (c) $A \subseteq A \cap B$. (d) $B \cap (A \cup B) = B$.
(e) $A \setminus (A \cap B) = A$.

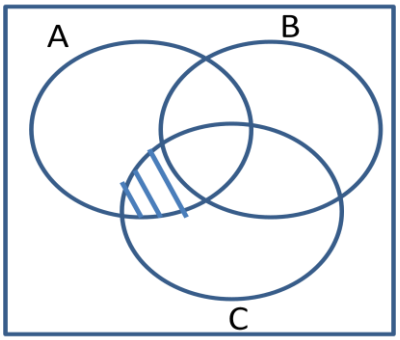
6) Let A and B be any two non-empty sets. If P(A) denotes the power set of A, which of the following is/are true?

(a) $A \cup B \in P(A \cup B)$. (b) $A \cap B \in P(A \cup B)$. (c) $|P(A)| = 2^{|A|}$.
(d) $A \cup B \subseteq P(A \cup B)$. (e) $A \cup B \in P(A \cap B)$.

7) Let A and B be two non-empty subsets of a universal set S. Which of the following is/are true?

(a) $A^c \cup B^c = (A \backslash B)^c \cap A$. (b) $(A \cup B)^c = (A^c \cap B^c)$. (c) $(A \cap B)^c = (A^c \cup B^c)$.
(d) $A^c \cap B^c = (A \backslash B)^c \cup B$. (e) $A^c \cap B^c = (B \backslash A)^c \cup B$.

8) Consider the following Venn diagram.



Which of the following sets is/are represented by the shaded portion?

(a) $(B \backslash C) \cap A$ (b) $(A \backslash B) \cap C$
(c) $(A \cap B \cap C)^c \setminus (A \cap B)$ (d) $(C \backslash B) \cap A$
(e) $(A \cap C) \setminus (A \cap B \cap C)$

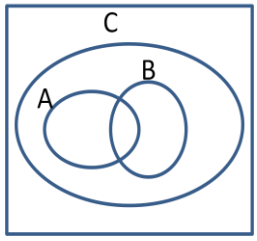
9) Let A,B and C be three sets. Which of the following is(are) correct?

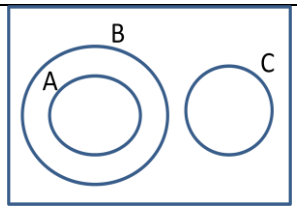
(a) $A \cup (B \cap C) = (A \cap B) \cup (A \cap C)$. (b) $A \cup (B \cup C) = (A \cup B) \cup C$.
(c) $A \cap (B \cup C) = A \cup (B \cap C)$. (d) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.
(e) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

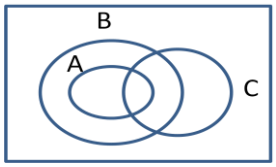
10) Let A and B be two sets. Which of the following is(are) correct?

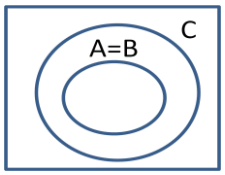
(a) $(A \subseteq B) \rightarrow (\forall x, x \in A \wedge x \in B)$. (b) $A = B \rightarrow A \subseteq B \vee B \subseteq A$.
(c) $(A \subseteq B) \rightarrow (\forall x, x \in A \rightarrow x \in B)$. (d) $A = B \rightarrow A \subseteq B \wedge B \subseteq A$.
(e) $(A \subseteq B) \leftrightarrow (\forall x, x \in A \rightarrow x \in B)$.

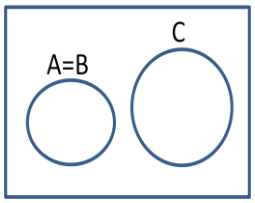
11) Let A,B and C be three non-empty sets such that $A \subseteq B$, $C \not\subset B$ and $A \cap C = \varnothing$. Which of the following Venn diagrams reflect these properties ?

(a) 

(b) 

(c) 

(d) 

(e) 

12) Consider the following truth table.

p	q	$(p \wedge q) \vee (p \rightarrow q)$
T	T	V1
T	F	V2
F	T	V3
F	F	V4

The truth values of V1, V2, V3 and V4 are

(a) T,T,T,T respectively. (b) T,T,T,F respectively. (c) T,T,F,T respectively.
(d) T,F,T,T respectively. (e) F,T,T,T respectively.

13) Let *p* and *q* be two atomic propositions. Which of the following are tautologies ?

(a) $(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$ (b) $p \vee (q \vee \sim p)$
(c) $\sim((p \rightarrow q) \leftrightarrow (\sim p \vee q))$ (d) $p \wedge (q \wedge \sim q)$
(e) $(p \rightarrow q) \leftrightarrow (\sim p \vee q)$

14) Consider the following truth table.

p	q	r	μ1	μ2	μ3
T	T	T	T	T	T
T	F	T	T	T	T
F	T	T	T	T	T
F	F	T	T	T	T
T	T	F	T	F	F
T	F	F	F	T	T
F	T	F	T	T	F
F	F	F	F	T	F

If $p \to (q \to r)$ is one of μ1, μ2, μ3 and $(p \to q) \to r$ is also one of μ1, μ2, μ3, which of the following is/are correct?

(a) $p \to (q \to r)$ is μ1 and $(p \to q) \to r$ is μ3.
(b) $p \to (q \to r)$ is μ3 and $(p \to q) \to r$ is μ2.
(c) $p \to (q \to r)$ is μ2 and $(p \to q) \to r$ is μ1.
(d) $p \to (q \to r)$ is μ2 and $(p \to q) \to r$ is μ3.
(e) $p \to (q \to r)$ is μ3 and $(p \to q) \to r$ is μ1.

15) Consider the following propositions.

(i) $p \wedge (p \vee q)$ (ii) p (iii) $p \wedge (\sim p \vee q)$ (iv) $p \wedge q$

Which of the above pairs of propositions are equivalent?

(a) (i) and (iii)
(b) (i) and (ii)
(c) (iii) and (iv)
(d) (ii) and (iv)
(e) (ii) and (iii)

16) Which of the following arguments is(are) valid?

(a) $p \to q, q \vdash p$ (b) $p \to q, p \vdash q$ (c) $p \to q, \sim q \vdash \sim p$
(d) $p \to q, \sim q \vdash p$ (e) $p \vdash p \vee q$

17) Consider the following sets of propositions.

(i) $p \wedge q, \sim p, q$ (ii) $p \to r,\ p,\ \sim r$ (iii) $(q \leftrightarrow p), \sim p, q$ (iv) $p \vee q, p, \sim q$

Which of the above sets of propositions are inconsistent?

(a) (i) and (ii) only (b) (ii) and (iii) only (c) (i), (iii) and (iv) only
(d) (iv) only (e) (i), (ii) and (iii) only

18) Let p and q be two atomic propositions. Which of the following propositions is(are) expressed in Canonical Disjunctive Normal Form?

(a) $(p \vee q \vee r) \wedge (p \vee \sim q \vee r) \wedge (p \vee \sim q \vee r)$
(b) $(p \wedge q \wedge r) \vee (p \wedge \sim q \wedge r) \vee (q \rightarrow p \wedge r)$
(c) $p \wedge q \wedge r$
(d) $p \vee q \vee r$
(e) $(p \wedge q \wedge r) \vee (p \wedge \sim q \wedge r) \vee (\sim p \wedge q \wedge r)$

19) Let p(x) be a predicate of the variable x defined on a domain D. Which of the following must be true so that the proposition $\exists x\ p(x)$ is false?

(a) There is a x_0 in D for which p(x) is false.
(b) For every x in D, p(x) is false.
(c) For every x in D, ~p(x) is true.
(d) There are no elements in D for which p(x) is true.
(e) $\forall x\ p(x)$ is false.

20) Let p(x) and q(x) be two predicates of the variable x defined by $x < 10$ and $x > 5$ respectively on N.

Which of the following propositions is(are) true?

(a) $\forall x\ p(x) \vee \forall x\ q(x)$ (b) $\forall x (p(x) \vee q(x))$ (c) $\exists x \sim p(x)$
(d) $\forall x \sim p(x) \wedge \forall x \sim q(x)$ (e) $\exists x (p(x) \wedge q(x))$

21) Suppose $x \in \{5, 10, 15, 20, 25\}$ and $y \in \{6, 12, 16, 24, 25\}$.

Which of the following propositions is(are) true?

(a) $\forall x\ \exists y\ x + y < 32$. (b) $\forall y\ \exists x\ x + y < 32$. (c) $\exists x\ \forall y\ x + y < 32$.
(d) $\exists x\ \exists y\ x + y < 32$. (e) $\forall x\ \forall y\ x + y < 32$.

22) Let p(x) and q(x) be two predicates of the variable x defined on a domain D. Consider the following proposition.

$\exists x (p(x) \vee q(x))$

Which of the following is/are equivalent to the above proposition?

(a) $\exists x\ p(x) \wedge \exists x\ q(x)$.
(b) $\exists x\ p(x) \vee \exists x\ q(x)$.
(c) $\sim \forall x (\sim p(x) \wedge \sim q(x))$.
(d) $\exists x (\sim q(x) \rightarrow p(x))$.
(e) $\exists x (\sim p(x) \rightarrow q(x))$.

23) Let R={(a,b), c} and S={(1,2), 3}. The Cartesian Product, $R \times S$ is equal to

(a) {(a,1),(a,2),(a,3),(b,1),(b,2),(b,3),(c,1),(c,2),(c,3)} .
(b) {((a,b,1,2),(c,3))} .
(c) {((a,b), (1,2)), ((a,b),3), (c, (1,2)), (c,3) } .
(d) $\varnothing$.
(e) {(1,a),(1,b),(1,c),(2,a),(2,b),(2,c),(3,a),(3,b),(3,c)}.

24) Let β be the relation defined by $\beta=\{(a,b)|\ a \in Z, b \in Z, a = b\}$.

Which of the following must be true?

(a) $\beta^{-1} = \beta$.
(b) β is not reflexive .
(c) β is symmetric .
(d) $D(\beta) \subseteq R(\beta)$.
(e) β is an equivalence relation .

25) Let A be the set of all living people and β be a relation defined on A by
$\alpha = \{(a,b) \mid b \text{ is a brother of } a\}$.

What is α^{-1}?

(a) $\alpha^{-1}=\{(a,b) \mid a \text{ is a brother of } b\}$.
(b) $\alpha^{-1}=\{(a,b) \mid (b,a) \in \alpha\}$.
(c) $\alpha^{-1}=\{(b,a) \mid a \text{ is a brother of } b\}$.
(d) $\alpha^{-1}=\{(a,b) \mid a \text{ is a sister of } b\}$.
(e) $\alpha^{-1}=\{(a,b) \mid b \text{ is a sister of } a\}$.

26) Suppose a relation ρ is non-empty and defined on a non-empty set X. Then ρ is said to be transitive if

(a) $\forall x \forall y \forall z\ (x,y) \in \rho \wedge (y,z) \in \rho \rightarrow (x,z) \in \rho$.
(b) $\forall x \forall y \forall z\ (x,y) \in \rho \wedge (z,y) \in \rho \rightarrow (x,z) \in \rho$.
(c) $\forall x \forall y \forall z\ (x,y) \in \rho \wedge (z,x) \in \rho \rightarrow (z,y) \in \rho$.
(d) $\exists x,\ x \in D(\rho) \wedge (x,x) \in \rho$.
(e) $\forall x \forall y \forall z\ (x,y) \notin \rho \vee (y,z) \notin \rho \vee (x,z) \in \rho$.

27) Let α and β be two relations defined on A={a,b,c} by
$\alpha=\{ (a,a), (a,b), (b,c), (c,a), (c,c)\}$
$\beta=\{(a,b), (a,c), (b,a), (c,c)\}$

Which of the following is/are true?

(a) $\alpha \mathbf{o} \beta = \{(a,b),(a,a),(b,c),(c,b),(c,c)\}$ and $\beta \mathbf{o} \beta=\{(a,a),(a,c),(b,b),(c,c)\}$.
(b) $\alpha \mathbf{o} \beta = \{(a,b),(a,a),(b,c),(c,b),(c,c)\}$ and $\beta \mathbf{o} \beta=\{(a,a),(a,c),(b,b),(b,c),(c,c)\}$.
(c) $\alpha \mathbf{o} \beta = (\ (a,c),(a,a),(c,c),\ (b,a),(b,b),(c,a)\}$ and $\beta \mathbf{o} \beta=\{(a,a),(a,c),(b,b),(b,c),(c,c)\}$.
(d) $\alpha \mathbf{o} \beta = \rho$.
(e) $\alpha \mathbf{o} \beta = \beta$.

28) Suppose an equivalence relation ρ defined on A={a,b,c,d,e} is given by

$\boldsymbol{\rho}$={ (a,a), (b,b), (c,c), (d,d), (e,e), (a,b), (b,a), (b,c), (c,b), (a,c), (c,a), (d,e), (e,d) }.

What is $[a]_\rho \cup [d]_\rho$?

(a) {b,c,d,e}	(b) {b,c,e}	(c) {a,d}
(d) A	(e) {a,b,c}	

29) Let R and F be the set of all relations and the set of all functions from A to B respectively. Which of the following is(are) true?

(a) $F \subseteq A \times B$.	(b) $R \subseteq A \times B$.	(c) $F \subset A \times B$.
(d) $R \subset A \times B$.	(e) $R = A \times B$.	

30) Let f be a 1-1 function and $x_1, x_2 \in D(f)$. Which of the following is(are) true?

(a) $D(f^{-1}) \subset R(f)$.
(b) $\forall x_1, \forall x_2 \quad x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$.
(c) $R(f^{-1}) = D(f)$.
(d) $R(f^{-1}) \subset D(f)$.
(e) $\forall x_1, \forall x_2 , f(x_1) = f(x_2) \Rightarrow x_1 = x_2$.

31) Let f be a function defined by $D(f)=\{x| x \in R \wedge x \neq 2\}$, $f(x) = (x-1)/(x-2)$ for $\forall x \in D(f)$. Which of the following is/are true?

(a) $D(f^{-1}) = \{ x| x \in R \wedge x \neq 1\}$, $f^{-1}(x) = (2x-1) / (x-1)$.
(b) $D(f^{-1}) = \{ x| x \in R \wedge x \neq 2\}$, $f^{-1}(x) = (x-1) / (x-2)$.
(c) $D(f^{-1}) = \{ x| x \in R \wedge x \neq 1/2\}$, $f^{-1}(x) = (x-1) / (2x-1)$.
(d) $D(f^{-1}) = \{ x| x \in R \wedge x \neq 1\}$, $f^{-1}(x) = (2x+1) / (x-1)$.
(e) $D(f^{-1}) = \{ x| x \in R \wedge x \neq -1/2\}$, $f^{-1}(x) = (x+1) / (2x+1)$.

32) Let the functions f and g be defined by $f(x) = x-1$ and $g(x) = x^2$ respectively where $x \in R$. Then $(f \boldsymbol{o} g)(x)$ is equal to

(a) $x^2(x-1)$.
(b) x^2.
(c) $x^2 - 1$.
(d) $x - 1$.
(e) $x^2 - 2x + 1$.

33) Let A={1,2} and B={3,4}. Let X = { f | f is a function from A **into** B } and Y={ g | g is a function from A **onto** B }. Which of the following is/are correct?

(a) n(X) = 8 and n(Y) = 2 .	(b) n(X) = 2 and n(Y) = 8 .
(c) n(X) = 4 and n(Y) = 2 .	(d) n(X) = 4 and n(Y) = 4 .
(e) n(X) = 2 and n(Y) = 4 .	

34) Let the 6-tuple <B,+,*, c,0,1> be a Boolean algebra where B is a set, + and * the sum and the product operators respectively, 0 and 1 the zero and the unit elements respectively and c the complement operator.

If a and b are elements of the set B, which of the following is/are correct?

(a) a + (a * b) = a.	(b) a * (a + b) = a.	(c) a + a = 0.
(d) a * a = 1.	(e) a * b + a * b^c = a.	

35) In how many ways can the first and second places be awarded to two persons from among 9 people?

(a) 36	(b) 72	(c) $\frac{9!}{7!}$	(d) $\frac{9!}{(7!)(2!)}$	(e) 9!

36) In how many ways can a committee of 5 be chosen from 10 people given that one Errick of them must be the chairman of the committee?

(a) $\frac{10!}{(4!)(6!)}$	(b) $\frac{10!}{(5!)(5!)}$	(c) $\frac{9!}{(4!)(5!)}$	(d) $\frac{9!}{(4!)}$	(e) $\frac{9!}{(5!)}$

37) If $B \subset A$ then $P(A|B)$ is equal to

(a) $P(B)$	(b) $\frac{P(B)}{P(A)}$	(c) $\frac{P(A)}{P(B)}$	(d) 0	(e) 1

Use the following information for Question 38 and 39.

A box contains four equal-sized tickets, numbered 1, 2, 3 and 4, and a second box contains three tickets also of the same size, numbered 4, 5, and 6. An experiment consists of selecting one ticket from the first box and then selecting one ticket from the second box.

38) The sample space for this experiment is

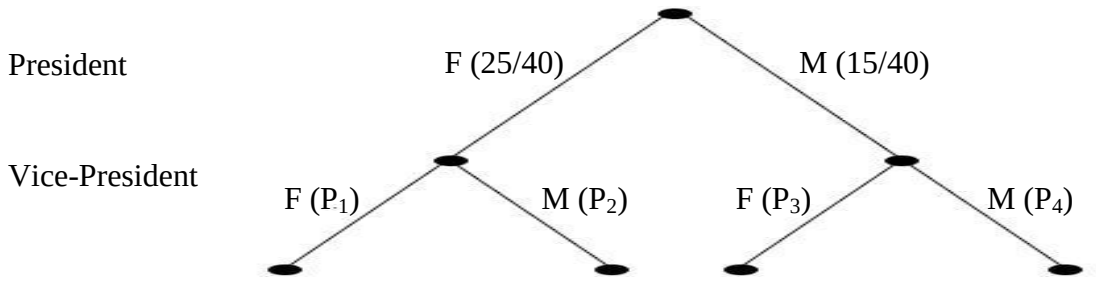
(a) {1, 2, 3, 4, 5, 6}. (b) {12}. (c) {(1, 2, 3, 4), (4, 5, 6)}.
(d) {(1,4), (1,5), (1,6) (2,4), (2,5), (2,6), (3,4), (3,5), (3,6), (4,4), (4,5), (4,6)}.
(e) {7}.

39) What is the probability that the sum of the numbers on the tickets chosen is at least 7?

(a) 9	(b) 3/4	(c) 3
(d) 1/4	(e) 0	

Use the following information for Question 40 and 41.

A company has 40 members, 25 of whom are females(F) and 15 of whom are males(M). The following tree diagram was prepared for the process of choosing a President and Vice-President for the company.



40) The probabilities P_1, P_2, P_3, and P_4 respectively are

(a) 24/39, 15/39, 25/39, 14/39.
(b) 25/40, 15/40, 25/40, 15/40.
(c) 24/40, 15/40, 25/40, 14/40.
(d) 25/39, 14/39, 24/39, 15/39.
(e) 25/39, 15/39, 25/39, 15/39.

41) What is the probability that both are males?

(a) 210/1560	(b) 225/1600	(c) 1145/1560	(d) 1200/1600	(e) 225/1521

42) Suppose the two events “high” and “low” make a disjoint partition of a sample space and “favourable” is any event. If P(high) = 0.3, P(low) = 0.7, P(favourable| high) = 0.9 and P(unfavorable| low) = 0.6, then P(favourable) is

(a) 0.10 .	(b) 0.27 .	(c) 0.30 .	(d) 0.55 .	(e) 0.63 .

43) As an accountant in your company, you classify 75% of your customers as "good credit" and the rest as "risky credit" depending on their credit rating. Customers in the "risky" category allow their accounts to go overdue 50% of the time on average, whereas those in the "good" category allow their accounts to become overdue only 10% of the time. What percentage of the overdue accounts are held by customers in the "risky credit" category?

(a) 20%	(b) 12.5%	(c) 62.5%	(d) 90%	(e) 93.7%

****